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MESON RESONANCES

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1. INTRODUCTION

In these lectures, we shall review the present experimental situation of meson states. We shall not try to cover the entire available information, but we shall concentrate our discussion on a few problems which seem for the moment to govern the activities of most of the experimental groups. Moreover, we shall limit ourselves essentially to the strong interaction aspects of the problem, therefore excluding topics such as leptonic decay modes of K mesons, and possible asymmetries in the $\pi^+\pi^-\pi^0$ decay mode of the η meson. In any case, the largest amount of work done in these fields does not apply directly to our subject. In these experiments, the K^0 and η mesons are used as tools, rather than studied for themselves.

It may be less justified to exclude from our discussion most of the work done on the electromagnetic decay modes of the meson states: the experimental information is usually not very rich in this field, and therefore does not lead to important conflicts of interpretation.

Even in the field of strong interactions, we shall be more concerned with the intrinsic properties of the meson states than with their production mechanism. This last subject would require, by itself, a series of lessons to be reasonably developed.

We shall first review briefly the basic techniques used in the determination of the properties of the meson states: conservation laws and selection rules, spin-parity analysis, Dalitz plot. We shall also mention some of the difficulties often encountered in the analysis of resonances, rescattering effects and Peierls' mechanism, interference effects, and Deck mechanism. Of course, these topics will not be studied in great detail: our survey is simply intended to allow everybody to follow the phenomenological description which will be presented on the meson states. For systematic discussions, we shall refer to specialized articles and review papers.

In the description of the experimental results, we shall try to keep as free as possible from any theoretical prediction.

2. SELECTION RULES

2.1 Conservation laws and quantum numbers

A particle is characterized by a mass M, a lifetime τ, a spin J, an electric charge number Q, a baryon number B, and two leptonic numbers L_μ and L_e.

For meson states, we have $B = L_\mu = L_e = 0$.

The strong interactions of mesons are consistent with the assumption that every meson has, in addition to the above quantum number, a definite intrinsic parity P, either positive or negative.

Instead of the lifetime τ, we shall often refer to the "width" Γ of the state ($\Gamma = 1/\tau$).

2.1.1 Isospin and strangeness. The mesons are observed to occur in multiplets; the members of a multiplet have approximately the same masses and the same interaction properties; they differ by their electric charge Q. For these reasons, it is useful to consider each member of a given multiplet as a charge state of a single particle: it introduces an additional quantum number, the isospin I [a multiplet of isospin I has (2I + 1) members]. To say that strong interactions are charge independent is equivalent to saying they are invariant under rotations in isospin space.

The analogy between isospin and ordinary spin is apparent. Whereas all rotations in ordinary space are physically meaningful, only certain rotations in isospin space have a physical meaning: this is because the charge of the physical states have to be integral multiples of the unit charge.

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If, for instance, I_z is related to the charge number Q, one may consider a rotation R of 180° around the y axis. Let us examine the effect of such a rotation for a doublet (think of the proton-neutron doublet):

The two eigenstates of the operator τ_z are $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

The operator τ_y which enters in the rotation R around the 0_y axis is identical to the corresponding Pauli operator:

$$\tau_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} .$$

The rotation R will transform the general isospinor ψ according to:

$$\psi \to \psi' = e^{i(\pi/2)\tau_y} \psi = i\, \tau_y\, \psi$$
$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

that is

$$\begin{aligned} p &\to -n \\ n &\to \ \ p , \end{aligned} \tag{1}$$

and the antinucleon isospinor will be transformed according to:

$$\bar{\psi} \to \bar{\psi}' = \bar{\psi} e^{-i(\pi/2)\tau_y} = \bar{\psi}(-i\, \tau_y) \quad ,$$

that is

$$\begin{aligned} \bar{p} &\to -\bar{n} \\ \bar{n} &\to \ \ \bar{p} . \end{aligned} \tag{2}$$

For the π-meson multiplet (I = 1), one may consider the bound states which are formed by the nucleon-antinucleon systems.

These systems can be reduced to a singlet:

$$|0,0\rangle = \frac{1}{\sqrt{2}} (\bar{p}p + \bar{n}n)$$

and a triplet:

$$\pi^+ = (p\bar{n})$$
$$\pi^o = \frac{1}{\sqrt{2}} (p\bar{p} - n\bar{n})$$
$$\pi^- = (n\bar{p}) .$$

Under the rotation R, applying formulae (1) and (2):

$$\begin{pmatrix} \pi^+ \\ \pi^o \\ \pi^- \end{pmatrix} \to - \begin{pmatrix} \pi^- \\ \pi^o \\ \pi^+ \end{pmatrix} \tag{3}$$

For "strange particles", I_z differs from Q by a constant which is characteristic of the multiplet: this constant is related to the "strangeness" of the meson multiplet:

$$Q = I_z + \frac{1}{2} S .$$

S and I_z are conserved in electromagnetic interactions as well as in strong interactions.

2.1.2 Charge conjugation and G parity. It has been found experimentally that to every meson characterized by the numbers

$$M, \Gamma, J, P, I, Q, \text{ and } S,$$

there is a meson with the numbers:

$$M, \Gamma, J, P, I, -Q, \text{ and } -S .$$

The members of these families have received the names of "particle" and "antiparticle"· by convention, the mesons with positive strangeness are called "particles". If S = 0, there is no way to distinguish between particle and antiparticle: the π^+ meson may be regarded as the particle π^+ or the antiparticle $\overline{\pi^-}$.

The operator C which transforms a particle state into its antiparticle state is called the charge conjugation operator:

$$C|X\rangle = |\bar{X}\rangle .$$

Strong interactions as well as electromagnetic interactions are assumed to be invariant under C.

For the neutral mesons with strangeness S = 0, the particle and antiparticle states are indistinguishable, i.e.

$$C = \pm 1 .$$

The C parity of neutral mesons, with strangeness S = 0, must be inferred from their electromagnetic or strong decay modes, the photon having C = -1 (the interaction between photons and charged particles is $j_\mu A_\mu$, where j_μ, the current, changes sign under C; therefore A_μ must also change sign, and the quantum of the field A_μ, the photon, has C = -1).

A system of n photons will have

$$C = (-1)^n.$$

For example, the observation of the decays

$$\pi^o \to \gamma + \gamma \quad \eta^o \to \gamma + \gamma$$

leads to

$$C(\pi^o) = +1 \quad C(\eta) = +1 . \tag{4}$$

For a multiplet of mesons with S = 0, only the neutral member is the eigenstate of C. To extend this symmetry to the charged members, one defines the G-parity:

$$G = C \cdot R, \tag{5}$$

C being the charge conjugation parity and R the rotation of 180° around the 0_y axis of the isospin space. The G-parity is therefore not conserved in electromagnetic interactions, and applies only to strong interactions.

The application of CR to the π meson multiplet, according to (3), leads to:

$$G \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix} \rightarrow - \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix} . \qquad (6)$$

The G-parity of the π mesons is thus negative. In general, the G-parity of a multiplet I of mesons, of which the neutral member has a charge conjugation parity C, is:

$$G = C(-1)^I . \qquad (7)$$

If a system is coupled strongly to sub-systems of G-parity, G_1, G_2, G_3, ..., its G-parity will be defined according to:

$$G = G_1 \cdot G_2 \cdot G_3 \ldots . \qquad (8)$$

For instance, the observation of $p \rightarrow \pi^+\pi^-$ leads to

$$G(\rho) = +1 .$$

2.2 Meson-antimeson pairs

The product of their intrinsic parities is 1, and the total parity of the system will be given by the parity of the relative angular momentum of the two objects:

$$P = (-1)^L . \qquad (9)$$

The charge parity C of a system of meson-antimeson will be given by

$$C = (-1)^{L+S} , \qquad (10)$$

where L is for the orbital angular momentum and S for the spin of the system.

This relation leads to the following:

$$G = (-1)^{L+S+I} . \qquad (11)$$

From Figs. (9) and (10), one deduces:

$$CP = (-1)^S . \qquad (12)$$

2.3 Applications to $(\pi\pi)$ and $(K\bar{K})$ systems:

For the $(\pi\pi)$ systems, we have:

$$P = (-1)^L, \; G = +1 = (-1)^{L+I} \quad \text{(L and I both even or odd)} . \qquad (13)$$

For the neutral system $(\pi^+\pi^-)$:

$$C = P = (-1)^L, \quad CP = +1 .$$

For the neutral system of two identical particles $(\pi^0\pi^0)$:

$$C = P = +1 \quad \text{(L and I must be even)} .$$

For the $(K\bar{K})$ system, we have

$$P = (-1)^L, \; G = (-1)^{L+I} . \qquad (14)$$

For the neutral $(K\bar{K})$ systems, we have also:

$$C = P = (-1)^L, \quad CP = +1 . \qquad (15)$$

Indeed, once the K^o and $\bar{K}^o$ have been produced in a strong interaction, the strong forces can be disregarded, and $K^o \leftrightarrow \bar{K}^o$ transitions are possible. If one assumes that CP is relatively well conserved, it is interesting to consider the eigenstates of CP. They are usually defined according to the following relations:

$$CP = +1 \; : \; |K_1^o\rangle = \frac{(K^o - \bar{K}^o)}{\sqrt{2}}$$

$$CP = -1 \; : \; |K_2^o\rangle = \frac{(K^o + \bar{K}^o)}{\sqrt{2}} \qquad (16)$$

with the convention:

$$CP|K^o\rangle = -|\bar{K}^o\rangle$$

$$CP|\bar{K}^o\rangle = -|K^o\rangle \qquad (17)$$

and:

$$|K^o\rangle = \frac{(K_1^o + K_2^o)}{\sqrt{2}}$$

$$|\bar{K}^o\rangle = \frac{(K_2^o - K_1^o)}{\sqrt{2}} \quad . \qquad (18)$$

Therefore, the C = +1 $(K^o\bar{K}^o)$ state will lead to:

$$|K^o\bar{K}^o\rangle = \frac{K^o\bar{K}^o + \bar{K}^oK^o}{\sqrt{2}}$$

$$= \frac{K_1^oK_2^o + K_2^oK_2^o - K_1^oK_1^o - K_2^oK_1^o}{2\sqrt{2}}$$

$$+ \frac{K_2^oK_1^o + K_2^oK_2^o - K_1^oK_1^o - K_1^oK_2^o}{2\sqrt{2}} = \frac{K_2^oK_2^o - K_1^oK_1^o}{\sqrt{2}} \qquad (19)$$

and no $K_1^oK_2^o$ will occur.
But for C = -1

$$|K^o\bar{K}^o\rangle = \frac{K^o\bar{K}^o - \bar{K}^oK^o}{\sqrt{2}}$$

$$= + \frac{K_1^oK_2^o + K_2^oK_2^o - K_1^oK_1^o - K_2^oK_1^o - K_2^oK_1^o - K_2^oK_2^o + K_1^oK_1^o + K_1^oK_2^o}{2\sqrt{2}} \qquad (20)$$

$$= \frac{K_1^oK_2^o - K_2^oK_1^o}{\sqrt{2}}$$

and no $K_1^oK_1^o$ nor $K_2^oK_2^o$ will occur.

According to Eq. (15):

$$CP = +1 \text{ and } P = (-1)^L ,$$

therefore

$$C = P = +1 \text{ for } (K_1^0K_1^0) \text{ and } (K_2^0K_2^0) \text{ systems} \quad (L \text{ must be even}); \qquad (21)$$

$$C = P = -1 \text{ for } (K_1^0K_2^0) \text{ system} \quad (L \text{ must be odd}). \qquad (22)$$

2.4 Nucleon-antinucleon

It is worth while to study briefly this system since many experiments, using the nucleon-anti-nucleon annihilations, have brought much interesting information about the mesons.

There are at least two other reasons for investigating this system: it may be possible, in the near future, that the annihilations be used to study the heavy mesons (with mass above 1.876 GeV) in "formation experiments type", as suggested by C. Peyrou in 1956, in analogy to the work done nowadays on the hyperons, with beams of K^- mesons.

Finally, the quark-antiquark system being, like the nucleon-antinucleon system, of the more general family of fermion-antifermion, our study of nucleons will apply to quarks and will be useful when we consider the quark model for the mesons.

For a fermion-antifermion state, the rules (9), (10), (11), (12) become:

$$P = (-1)^{L+1} \qquad (23)$$

$$C = (-1)^{L+S} \qquad (24)$$

$$G = (-1)^{L+S+I} \qquad (25)$$

$$CP = (-1)^{S+1} \; . \qquad (26)$$

C applies only, of course, to the neutral systems ($p\bar{p}$) and ($n\bar{n}$). For the charged systems ($\bar{p}n$) and ($p\bar{n}$), one can still apply the G-parity. Table 1 gives the quantum numbers of nucleon-antinucleon pairs for L = 0 and L = 1. For each spin-parity state, we have to consider the two possibilities for isospin I = 0, I = 1. The C number refers to the neutral members of the multiplet.

In Table 1, we have also indicated the lightest meson which shares the quantum numbers of the nucleon-antinucleon system, when it is known. We have also indicated the possible ($K^0\bar{K}^0$) decay in this last column, P is for "forbidden by parity".

The observation of the annihilation $\bar{p}p \to K_1^0K_2^0$ and the absence of annihilation $\bar{p}p \to K_1^0K_1^0$ at rest, has been taken for an experimental verification by Armenteros et al.[1]) of the argument of Day, Snow and Sucher[2]), and d'Espagnat[3]) that stopped $\bar{p}$ are captured in S-states in liquid hydrogen.

This result will be extensively used for the analysis of stopped $\bar{p}p$ annihilations.

Table 1

Spectroscopic notation	L	S	J^P	C	I^G	Meson representation	$(K^0\bar{K}^0)$
$^1S_0$	0	0	0^-	+	0^+	η	P
					1^-	π	
$^3S_1$	0	1	1^-	-	0^-	ω	$K_1^0K_2^0$
					1^+	ρ	
$^1P_1$	1	0	1^+	-	0^-		P
					1^+		
$^3P_0$	1	1	0^+	+	0^+		$K_1^0K_1^0$
					1^-		$K_2^0K_2^0$
$^3P_1$	1	1	1^+	+	0^+		P
					1^-		
$^3P_2$	1	1	2^+	+	0^+	f	$K_1^0K_1^0$
					1^-	A_2	$K_2^0K_2^0$

3. SPIN-PARITY ANALYSIS

3.1 Pure and mixed states. Density matrix. Polarization. Alignment.

A meson may be produced in a pure state or in a mixed state. In the case of a mixed state, the average value of a physical quantity Q will be:

$$\langle Q \rangle = \sum_i w_{(i)} \langle Q^{(i)} \rangle \tag{27}$$

with

$$\langle Q^{(i)} \rangle = \langle \psi^{(i)} | Q | \psi^{(i)} \rangle \ . \tag{28}$$

Let us define the pure states $|\psi^{(i)}\rangle$ in terms of the eigenfunctions of the angular momentum: for a given momentum j there are (2j + 1) eigenfunctions corresponding to the different values of the magnetic quantum number m:

$$|\psi^{(i)}\rangle = \sum_m a_m^{(i)} |\psi_m\rangle \tag{29}$$

replacing in Eq. (28)

$$\langle Q^{(i)} \rangle = \sum_{m,m'} Q_{mm'} \, a_m^{(i)*} \, a_{m'}^{(i)} \tag{30}$$

with

$$Q_{mm'} = \langle \psi_m | Q | \psi_{m'} \rangle \tag{31}$$

and

$$\langle Q \rangle = \sum_{m,m'} Q_{mm'} \, \rho_{m'm} \tag{32}$$

with

$$\rho_{m'm} = \sum_i w_{(i)} \, a_m^{(i)*} \, a_{m'}^{(i)} \ . \tag{33}$$

The $Q_{mm'}$ are independent of the pure states $|\psi^{(i)}\rangle$ and of the weights $w_{(i)}$; these independences are regrouped in the

density matrix elements $\rho_{m'm}$.

Equation (32) may also be written:

$$\langle Q \rangle = \sum_m (Q\rho)_{mm} = \mathrm{Tr}\,(Q\rho) \ . \tag{34}$$

This basic relation between the average value of some physical quantity Q and the density matrix may be used either to determine the density matrix elements $\rho_{mm'}$, starting from the measurements of the average values $\langle Q \rangle$, or to evaluate the average value of any physical quantity characterizing the system, knowing ρ.

We shall make use of the measurement of the angular distributions to determine the $\rho_{mm'}$'s, which in turn can be used to specify the angular momentum of the decaying system (see Section 3.3).

Since every element of ρ may, in principle, be complex, the complete determination of ρ needs $2(2j + 1)^2$ measurements; however, the density matrix is Hermitian [the average, Eq. (34), must be real], therefore:

$$\rho_{m'm'} = \rho^*_{mm'} \quad , \tag{35}$$

and the total probability to find the system in any state must be 1, that is,

$$\mathrm{Tr}\ \rho = 1 \quad . \tag{36}$$

These two conditions reduce the number of measurements to $(2j + 1)^2 - 1 = 4j(j + 1)$, which can be compared to the $2(2j + 1) - 2 = 4j$ numbers necessary to specify a pure state (the minus 2 comes from the normalization and an over-all unobservable phase).

The polarization is given in terms of the expectation value for the spin, according to Eq. (34):

$$\vec{p} \sim \mathrm{Tr}\ (\vec{S}\rho) \quad . \tag{37}$$

There is no polarization if the population of the states with z-component of the spin m equals the population with component -m.

When the polarization is zero, there is still the possibility of an alignment, if the populations of the states of different m are different.

For example, for a spin-one particle, if we consider the diagonal representation of ρ in the m basis,

$$\rho = \begin{pmatrix} \rho_{11} & 0 & 0 \\ 0 & \rho_{00} & 0 \\ 0 & 0 & \rho_{-1-1} \end{pmatrix} . \tag{38}$$

The polarization is zero if $\rho_{11} = \rho_{-1-1}$, and the alignment is zero if $\rho_{00} = \frac{1}{2}(\rho_{11} + \rho_{-1-1})$.

It is therefore advantageous to select the production of events for the spin-parity analysis, in order to maximize the polarization or the alignment of the meson under study: this selection is achieved, in Adair's analysis, by concentrating on the forward and backward directions of the particle to be studied with respect to the incident beam.

3.2 Adair's analysis

Let us consider the process:

$$a + b \rightarrow c + d \quad , \tag{39}$$

where a is the beam particle with spin 0 (π or K meson);

b is the target particle with spin $\frac{1}{2}$ (nucleon) unpolarized;

c is the recoil particle with spin λ; and

d is a meson of spin J, to be determined.

In the forward direction, the production amplitude A^m will be:

$$A^m = \sum_{\mu} \sum_{L} a_{L\mu} \, \psi_\lambda^\mu \, \psi_J^{m-\mu} \, Y_L^o \quad . \tag{40}$$

Since the orbital angular momentum for forward production has no component in the beam direction,

ψ_λ^μ is the spin wave function for the recoil particle c,

$\psi_J^{m-\mu}$ the spin wave function for the meson d, m being the magnetic quantum number of the target particle b along the beam axis.

If the target is made of unpolarized protons, the meson d is produced in a mixed state made of an incoherent mixture of pure states with $m = \frac{1}{2}$ and $m = -\frac{1}{2}$ with equal weights. When $\lambda = \frac{1}{2}$, one has to consider the "non-spin-flip" amplitude: $m-\mu = 0$, and the "spin-flip" amplitude: $m-\mu = 1$.

If the meson decays into two pseudoscalar mesons, its decay angular distribution in its centre of mass will be:

$$W(\vartheta) = |a \, Y_J^o|^2 + |b \, Y_J^1|^2 \tag{41}$$

where Y_J^o and Y_J^1 are the spherical harmonics, corresponding to the non-spin-flip amplitude and the spin-flip amplitude, respectively (the meson d being produced in the forward direction, its decay is independent of φ).

For example, consider the reaction

$$K^- + p \to K^* + p \quad .$$

For the forward emitted K^*, one expects decay angular distribution, with respect to the beam direction, of the type

$$W(\vartheta) = a \cos^2 \vartheta + b \sin^2 \vartheta \quad . \tag{42}$$

If there is no spin flip (as must be the case if the reaction takes place via pion exchange), $W(\vartheta) = a \cos^2 \vartheta$.

If the meson d decays into a spin-one and a spin-zero particle, and is completely aligned at production (described by a spin wave function of the type ψ_J^o), its decay angular distribution will depend not only on its spin, but also on its parity: if its parity P is given in terms of its spin J and of the product of the intrinsic parities P′ of the decay products by:

$$P = P' \, (-1)^J \quad , \tag{43}$$

then the orbital angular momentum to be introduced between the decay products is unique and the decay angular distribution is well defined:

$$W(\vartheta) = |Y_J^1|^2 \tag{44}$$

$$J = 1 \quad W(\vartheta) \sim \sin^2 \vartheta$$

$$J = 2 \quad W(\vartheta) \sim \sin^2 2\vartheta \quad .$$

If the parity P of the meson d is given by the relation:

$$P = P'(-1)^{J+1} \quad , \qquad (45)$$

the orbital angular momentum to be introduced between the decay products can take two values, and the decay angular distribution does not reduce to a simple expression. However, if the available energy in the decaying system is small, centrifugal barriers may favour the lower angular momentum.

For example, consider the decay:

$$D \rightarrow K^* + \bar{K} \quad . \qquad (46)$$

Since the mass of the D meson is 1290 MeV, this decay is only possible when one takes into account the width of the $K^*(891)$, In these conditions, if the spin-parity of the D to be tried is of the series 1^+, 2^- ..., it is likely that the orbital angular momentum to be considered will be mainly 0, 1 ..., although 2, 3 ... are allowed by the spin-parity assignments.

The decay angular distribution will take the form:

$$\begin{array}{ll} J^P = 0^- & W(\vartheta) = 1 \\ \quad\; 1^+ & \qquad\quad 1 \\ \quad\; 2^- & \qquad\quad 1 + 3\cos^2\vartheta \; . \end{array}$$

Let us close this discussion on the Adair analysis with the following example:

Chinowsky et al.[4] have observed that the reaction:

$$K^- + p \rightarrow p + K^+ + \pi^- + \pi^+ \qquad (47)$$

goes predominantly via

$$K^- + p \rightarrow K^{*o} + N^{*++} \qquad (48)$$

at 1.9 GeV, and that the K^{*o} are strongly peaked in the forward direction. This peaking favours Adair's analysis of the reaction (48), both for the $K^*(891)$ and the N^* (which are, of course, peaked backward).

Denoting by m_p, m_{K^*}, m_{N^*} the projections of the spins of the proton, K^*, and N^* along the beam direction, the decay angular distributions $W_{K^*}(\vartheta)$ and $W_{N^*}(\vartheta)$ are given in Table 2 where one considers the different possible pure states for $J^P(K^*) = 0^+, 1^-, 2^+$.

Of course, K* and N* are not necessarily produced in a pure state, and intermediate angular distributions are always possible.

Experimentally, one finds

$$W_{K^*}(\vartheta) = \cos^2\vartheta, \qquad W_{N^*}(\vartheta) = 1 + 3\cos^2\vartheta \quad .$$

This is in agreement with $J^P(K^*) = 1^-$ when the K* is completely aligned ($m_{K^*} = 0$).

This is, of course, taken for direct evidence in favour of $J^P(K^*) = 1^-$, the complete alignment being explained, in this case, by the predominant contribution of the one-pion exchange mechanism to the production:

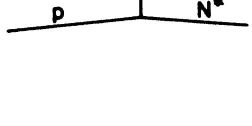


Table 2

$J^P(K^*)$	m_p	m_{K^*}	m_{N^*}	$W_{K^*}(\vartheta)$	$W_{N^*}(\vartheta)$
0^+	$\frac{1}{2}$	0	$\frac{1}{2}$	1	$1 + 3\cos^2\vartheta$
1^-	$\frac{1}{2}$	1	$-\frac{1}{2}$	$\sin^2\vartheta$	$1 + 3\cos^2\vartheta$
	$\frac{1}{2}$	0	$\frac{1}{2}$	$\cos^2\vartheta$	$1 + 3\cos^2\vartheta$
	$\frac{1}{2}$	-1	$\frac{3}{2}$	$\sin^2\vartheta$	$\sin^2\vartheta$
2^+	$\frac{1}{2}$	2	$-\frac{3}{2}$	$\sin^4\vartheta$	$\sin^2\vartheta$
	$\frac{1}{2}$	1	$-\frac{1}{2}$	$\sin^2 2\vartheta$	$1 + 3\cos^2\vartheta$
	$\frac{1}{2}$	0	$\frac{1}{2}$	$(3\cos^2\vartheta - 1)^2$	$1 + 3\cos^2\vartheta$
	$\frac{1}{2}$	-1	$+\frac{3}{2}$	$\sin^2 2\vartheta$	$\sin^2\vartheta$

This mechanism is also supported by the strong peaking of the K^* in the forward direction. (The π meson being a spin-zero particle, the spin wave function of the K^* cannot carry a spin along the beam direction, thence its complete alignment.)

3.3 More general treatment, using the density matrix

3.3.1 Two-body decay. Call $A_m(\vec{p},\lambda_1,\lambda_2)$ the decay amplitude for

$$B \to 1 + 2 \tag{49}$$

from a definite state (j,m) to the state $|\vec{p},\lambda_1,\lambda_2\rangle$ where $\vec{p}$ refers to the momentum of particle 1 in the B centre of mass, λ_1 and λ_2 are the helicities of 1 and 2 [Jacob and Wick[5], Berman and Jacob[6]]:

$$A_m(\vec{p},\lambda_1,\lambda_2) = \langle \vec{p},\lambda_1,\lambda_2 | U | j,m\rangle \ . \tag{50}$$

U is invariant under rotations and reflections: $U = U(\lambda_1,\lambda_2)$. The requirement of parity conservation leads to

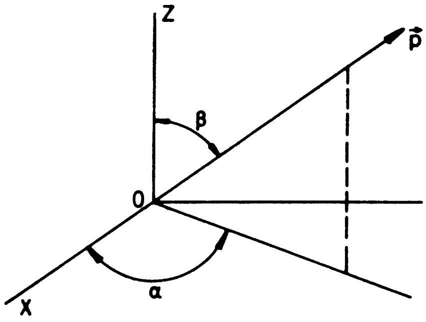
$$U(-\lambda_1,-\lambda_2) = \eta_1,\eta_2,\eta_B(-1)^{j-s_1-s_2}\, U(\lambda_1,\lambda_2) \ , \tag{51}$$

where η_1,η_2,η_B refer to the intrinsic parity of particles 1, 2, B;
s_1,s_2 refer to the spin of particles 1, 2.

For a resonant state B described by a density matrix ρ, the angular distribution of the decay is then:

$$W(\alpha,\beta) = \text{const} \sum_{m,m'} \sum_{\lambda_1,\lambda_2} A_m(\vec{p},\lambda_1,\lambda_2)\, \rho_{mm'} A^*_{m'}(\vec{p},\lambda_1,\lambda_2) \ , \tag{52}$$

where α and β refer to the polar angles of the vector $\vec{p}$ with respect to a standard reference frame



The introduction of the $D^j_{m\lambda}$ functions in the expression of the decay amplitudes transforms the expression of $W(\alpha,\beta)$ into:

$$W(\alpha,\beta) = \text{const} \sum_{m,m',\lambda_1,\lambda_2} |U(\lambda_1,\lambda_2)|^2 D^{j*}_{m\lambda}(\alpha,\beta,0) D^j_{m'\lambda}(\alpha,\beta,0)\rho_{mm'} \tag{53}$$

$$\lambda = \lambda_1 - \lambda_2 \quad .$$

<u>Example:</u> two-body decay of a spin-one particle decaying into two spin-zero particles ($\rho \to \pi\pi$, $K^* \to K\pi$ ...)

$$U(\lambda_1,\lambda_2) = \text{const}$$

$$\begin{aligned} W(\alpha,\beta) = \frac{3}{8\pi} \Big\{ & 2\,\rho_{00} \cos^2\beta + (\rho_{11} + \rho_{-1-1}) \sin^2\beta \\ & + 2 \sin^2\beta\, (\sin 2\alpha \,\text{Im}\, \rho_{1-1} - \cos 2\alpha \,\text{Re}\, \rho_{1-1}) \\ & + \sqrt{2} \sin 2\beta\, (\sin\alpha \,\text{Im}\, \rho_{10} - \cos\alpha \,\text{Re}\, \rho_{10}) \\ & + \sqrt{2} \sin 2\beta\, (\sin\alpha \,\text{Im}\, \rho_{-10} + \cos\alpha \,\text{Re}\, \rho_{-10}) \Big\} . \end{aligned} \tag{54}$$

This expression may be simplified by a suitable choice of the quantification axis OZ: if, for instance, OZ is chosen to be the normal to the production plane, then

$$\rho_{m'm} = 0 \text{ if } m\text{-}m' \text{ odd;}$$

for a spin-one particle, the density matrix reduces to:

$$\begin{pmatrix} \rho_{11} & 0 & \rho_{1-1} \\ 0 & \rho_{00} & 0 \\ \rho^*_{1-1} & 0 & \rho_{-1-1} \end{pmatrix} . \tag{55}$$

If OZ is chosen to be along the resonance momentum,

$$\rho_{-m-m'} = (-1)^{m-m'} \rho_{mm'} ,$$

and for a spin-one particle, the density matrix reduces to:

$$\begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1-1} \\ \rho^*_{10} & \rho_{00} & -\rho^*_{10} \\ \rho_{1-1} & -\rho_{10} & \rho_{11} \end{pmatrix} \tag{56}$$

with three real elements (ρ_{11}, ρ_{00}, ρ_{1-1}) and one complex element ρ_{10}. With this choice of OZ, the angular distribution [Eq. (54)] reduces to:

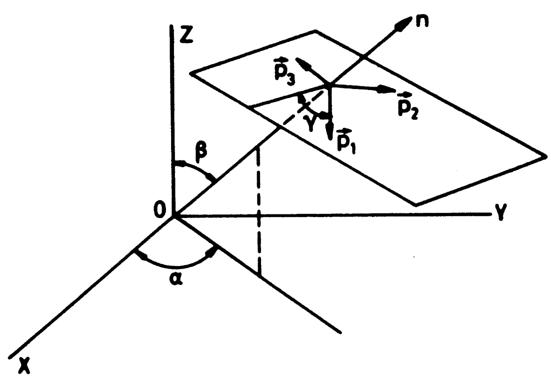
$$W(\alpha,\beta) = \frac{3}{4\pi} \left\{ \rho_{00} \cos^2 \beta + \rho_{11} \sin^2 \beta - \rho_{1-1} \sin^2 \beta \cos 2\alpha - \sqrt{2} \operatorname{Re} \rho_{10} \sin 2\beta \cos \alpha \right\} . \tag{57}$$

3.3.2 Three-body decay

$$B \to 1 + 2 + 3 .$$

The nine variables which define the system ($\vec{p}_1$, $\vec{p}_2$, $\vec{p}_3$, each momentum being defined in terms of the energy E_i and two angles ϑ_i, φ_i) reduce to five independent variables once the four-momentum energy conservation equations are entered.

One can choose for these five variables two of the three energies E_1, E_2, E_3, and three angles α, β, γ, defining the normal to the decay plane (α, β) and the direction of one particle in this plane



The study of the distribution of the energies is usually done on the Dalitz plot (see Section 3.5). We are now interested in the angular distribution of the normal to the decay plane.

The decay amplitude is written:

$$A_m(E_i,\lambda_i;\alpha,\beta,\gamma) = \langle E_i,\lambda_i;\alpha,\beta,\gamma|U|j,m\rangle \quad , \tag{58}$$

where j,m refer to the angular momentum of the decay system;

α,β,γ are the angles defined above;

E_i,λ_i (i = 1,2,3) are the energies and helicities of the three particles 1,2,3.

The angular distribution $W(\alpha,\beta)$ of the normal to the decay plane for a state described by a density matrix ρ will be:

$$W(\alpha,\beta) = \text{const} \sum_{m,m',\lambda_1,\lambda_2,\lambda_3} \int dE_1\, dE_2\, d\gamma\, A_m(E_i,\lambda_i;\alpha,\beta,\gamma)\rho_{mm'}A^*_{m'}(E_i,\lambda_i;\alpha,\beta,\gamma) \quad . \tag{59}$$

When the $D^j_{mk}(\alpha,\beta,\gamma)$ are introduced:

$$W(\alpha,\beta) = \text{const} \sum_{m,m',\lambda_1,\lambda_2,\lambda_3} \sum_k \left|U_k(\lambda_i)\right|^2 D^{j*}_{mk}(\alpha,\beta,0)D^j_{m'k}(\alpha,\beta,0)\rho_{mm'} \tag{60}$$

with

$$\left|U_k(\lambda_i)\right|^2 = \int dE_1\, dE_2 \quad \left|\langle E_i,\lambda_i;j,m,k|U|j,m\rangle\right|^2 \quad .$$

We now have several amplitudes, instead of one, for the two-body decay; their number and the actual values for k depend on the spin-parity of the decay system:

<u>Table 3</u>

Spin-parity of B	No. of amplitudes	Actual k values
0^+	0	
0^-, 1^-	1	0
1^+, 2^+	2	1, -1
2^-, 3^-	3	2, 0, -2

<u>Example</u>: <u>decay into three pseudoscalar mesons.</u>

For a spin-parity 1^- meson, we have only one amplitude and again get the distribution (54) for the normal to the decay plane. Formula (54) reduces again to formula (57) if OZ is taken along the resonance B momentum.

For a spin-parity 1^+ meson, we have two amplitudes U_1 and U_{-1}. If we write:

$$\lambda = \frac{|U_1|^2 - |U_{-1}|^2}{|U_1|^2 + |U_{-1}|^2}$$

we get

$$\begin{aligned} W(\alpha,\beta) \sim\ & (\rho_{11} + \rho_{-1-1}) \frac{1 + \cos^2\beta}{2} + \rho_{00} \sin^2\beta \\ & + \frac{1}{\sqrt{2}} \sin 2\beta \left[(\mathrm{Re}\ \rho_{10} - \mathrm{Re}\ \rho_{-10}) \cos\alpha - (\mathrm{Im}\ \rho_{10} + \mathrm{Im}\ \rho_{-10}) \sin\alpha \right] \\ & + \sin^2\beta\ (\cos 2\alpha\ \mathrm{Re}\ \rho_{1-1} - \sin 2\alpha\ \mathrm{Im}\ \rho_{1-1}) \qquad (61) \\ & + \lambda \left\{ (\rho_{11} - \rho_{-1-1}) \cos\beta + \sqrt{2} \sin\beta \left[(\mathrm{Re}\ \rho_{10} + \mathrm{Re}\ \rho_{-10}) \cos\alpha \right.\right. \\ & \left.\left. - (\mathrm{Im}\ \rho_{10} - \mathrm{Im}\ \rho_{-10}) \sin\alpha \right] \right\} . \end{aligned}$$

If OZ is taken along the resonance B momentum, formula (61) reduces to:

$$\begin{aligned} W(\alpha,\beta) \sim\ & \rho_{11}(1 + \cos^2\beta) + \rho_{00} \sin^2\beta + \sqrt{2} \sin 2\beta \cos\alpha\ \mathrm{Re}\ \rho_{10} + \rho_{1-1} \sin^2\beta \cos 2\alpha \\ & - \lambda\ 2\sqrt{2}\ \mathrm{Im}\ \rho_{10} \sin\alpha \sin\beta\ . \qquad (62) \end{aligned}$$

Example: $H \to \rho\pi \to \pi^+\pi^-\pi^0$.

Let us consider as a final example a meson H which decays into three pseudoscalar mesons π through a ρ meson.

One may consider the angle β, measured in the H centre of mass, between the beam and the normal to the H decay plane;

φ, measured also in the H centre of mass, between the beam and the ρ meson;

ϑ, measured in the ρ centre of mass, between the beam and one of the pions of the ρ decay.

If these angular distributions are not all flat, it rules out $J^P = 0^-$ for the H meson. $J^P = 0^+$ is excluded by spin-parity arguments ($0^+ \not\to 0^-0^-0^-$).

To disentangle $J^P = 1^-$ from $J^P = 1^+$, consider the angular distributions $W(\alpha,\beta)$ given above [formulae (54) and (61)] once the integration over α is performed:

$$J^P = 1^-:\ (54) \to W(\beta) \sim 2\ \rho_{00} \cos^2\beta + (\rho_{11} + \rho_{-1-1})\ (1 - \cos^2\beta)$$

$$\text{but } \rho_{00} + \rho_{11} + \rho_{-1-1} = 1\ ,$$

$$\text{therefore:}\quad W(\beta) \sim 1 + \frac{3\rho_{00} - 1}{1 - \rho_{00}} \cos^2\beta\ ; \qquad (63)$$

$$J^P = 1^+:\ (61) \to W(\beta) \sim \rho_{00}(1 - \cos^2\beta) + (\rho_{11} + \rho_{-1-1}) \frac{1 + \cos^2\beta}{2}$$

$$\text{therefore: } W(\beta) \sim 1 + \frac{1 - 3\,\rho_{00}}{1 + \rho_{00}} \cos^2 \beta \;. \qquad (64)$$

For the angular distribution $W(\varphi)$, we have to sum over the ρ-meson spin states:

$$W(\varphi) \sim \rho_{11} \, |S_1^1|^2 + \rho_{00} \, |S_1^0|^2 + \rho_{-1-1} \, |S_1^{-1}|^2$$

where the S's are the spin amplitudes of the H meson.

If $J^P(H) = 1^-$, they may be written:

$$\begin{aligned} S_1^1 &= Y_1^1(\varphi)\psi_1^0 - Y_1^0(\varphi)\psi_1^1 \\ S_1^0 &= Y_1^1(\varphi)\psi_1^{-1} - Y_1^{-1}(\varphi)\psi_1^1 \\ S_1^{-1} &= Y_1^0(\varphi)\psi_1^{-1} - Y_1^{-1}(\varphi)\psi_1^0 \end{aligned}$$

where the ψ's are the spin functions of the ρ meson.

Using again: $\rho_{11} = \rho_{-1-1}$ and $\rho_{11} = (1-\rho_{00})/2$, the angular distribution $W(\varphi)$ is of the form:

$$W(\varphi) \sim 1 + \frac{1 - 3\,\rho_{00}}{1 + \rho_{00}} \cos^2 \varphi \;. \qquad (65)$$

If $J^P(H) = 1^+$ we get for the simplest configuration (S-wave between the ρ and the π mesons)

$$W(\varphi) \sim 1 \;. \qquad (66)$$

Similarly, we get for $W(\vartheta)$,

$$J^P(H) = 1^- \quad W(\vartheta) \sim 1 + \frac{1 - 3\,\rho_{00}}{1 + \rho_{00}} \cos^2 \vartheta \qquad (67)$$

$$J^P(H) = 1^+ \quad W(\vartheta) \sim 1 + \frac{3\,\rho_{00} - 1}{1 - \rho_{00}} \cos^2 \vartheta \;. \qquad (68)$$

These three angular distributions may be compared with the experimental data. As we have seen, for spin less than 2, they depend on one parameter: ρ_{00}.

3.4 Zemach analysis

It is customary to describe states of angular momentum ℓ, with the magnetic quantum number m, by spherical harmonics $Y_\ell^m(\vartheta,\varphi)$.

It is more convenient to work with a tensor representation; this technique has been extensively developed by Zemach[7)].

For a system of two spinless mesons, characterized by the momentum $\vec{p}$ in the centre of mass: $\vec{p} = (p,\vartheta,\varphi)$, one may form tensors of any rank:

$$T_0 = 1 \qquad T_i = p_i \qquad T_{ij} = p_i p_j \;\ldots$$

(the p_i's being the Cartesian coordinates of $\vec{p}$).

A symmetric and traceless tensor of rank ℓ, such as the spherical harmonics $Y_\ell^m(\vartheta,\varphi)$, provides a $(2\ell + 1)$ dimensional representation of the rotation group.

For instance, in the scattering of two spinless mesons, the matrix element will be written:

$$\langle f|M^{\ell}|i\rangle \sim f(p^2) \sum_{i,j,k\ldots} T^{\ell}_{ijk\ldots}(\vec{p}_i)\, T^{\ell}_{ijk\ldots}(\vec{p}_f) \qquad (69)$$

with:

$$\begin{aligned} M^0 &\sim 1 \\ M^1 &\sim \vec{p}_i \cdot \vec{p}_f \\ M^2 &\sim (\vec{p}_i \cdot \vec{p}_f)^2 - \frac{1}{3}\, p_i^2 p_f^2 \\ M^3 &\sim (\vec{p}_o \cdot \vec{p}_f)^3 - \frac{3}{5}\, p_i^2 p_f^2\, (\vec{p}_i \cdot \vec{p}_f) \quad . \end{aligned} \qquad (70)$$

To give a complete description of the system, we must introduce the isospin coordinates as well, and symmetrize the over-all expression.

Let us consider the reaction:

$$\begin{matrix} \bar{p}p \to \rho\pi \to & \pi_1 & \pi_2 & \pi_3 \\ & \vec{a} & \vec{b} & \vec{c} \end{matrix} \qquad (71)$$

with $\bar{p}$ at rest.

The over-all transition matrice M will be decomposed in its isospin part M_I and its spin-parity component M_J:

$$M = M_I \cdot M_J \quad .$$

3.4.1 Isospin M_I: Let us describe the three π mesons by three isospin vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ (the charge states are related to the Cartesian coordinates of these vectors) by:

$$a_{\pm} = \mp \frac{1}{\sqrt{2}} (a_x \pm i a_y)$$

$$a_0 = a_z \quad .$$

According to whether the system (π_1, π_2) is I = 0, 1, 2, we shall write its isospin part:

$$\begin{aligned} I = 0 \quad & \vec{a} \cdot \vec{b} \\ I = 1 \quad & \vec{a} \times \vec{b} \\ I = 2 \quad & \frac{1}{2}(a_i b_j + a_j b_i) - \frac{1}{3}\,\delta_{ij}(\vec{a} \cdot \vec{b}) \qquad \text{(I: unit matrix)} \ . \end{aligned} \qquad (72)$$

Since we want (π_1, π_2) to form a ρ meson, we choose

$$\vec{a} \times \vec{b} \quad .$$

The composition of this isospin with $\vec{c}$ will depend on the isospin of the initial state:

$$\begin{aligned} I(\bar{p}p) = 0 \ &: \ (\vec{a} \times \vec{b}) \cdot \vec{c} \\ I(\bar{p}p) = 1 \ &: \ (\vec{a} \times \vec{b}) \times \vec{c} \quad . \end{aligned} \qquad (73)$$

3.4.2 Spin-parity M_J: $(\vec{a} \times \vec{b}) \cdot \vec{c}$ being antisymmetric in the exchange of any two particles, M_J will have to be completely antisymmetric and M will be written for $I(\bar{p}p) = 0$:

$$(\vec{a} \times \vec{b}) \cdot \vec{c} \;\; A(1,2,3) \quad , \tag{74}$$

A(1,2,3) being antisymmetric in the exchange of any two particles.

For $I(\bar{p}p) = 1$, the space function must be antisymmetric in the exchange of $\vec{a}$ and $\vec{b}$, for the term $(\vec{a} \times \vec{b}) \times \vec{c}$, and three such terms must be considered:

$$I(\bar{p}p) = 1 \;\; : \;\; M = + [(\vec{a} \times \vec{b}) \times \vec{c}]B(1,2) + [(\vec{b} \times \vec{c}) \times \vec{a}]B(2,3) + [(\vec{c} \times \vec{a}) \times \vec{b}]B(3,1) \tag{75}$$

$$= (\vec{a} \cdot \vec{b})\vec{c}[A(2,3) - B(3,1)] + (\vec{b} \cdot \vec{c})\vec{a}[B(3,1) - B(1,2)] + (\vec{c} \cdot \vec{a})\vec{b}[B(1,2) - B(2,3)] \tag{75'}$$

With the ρ mesons we form a space vector: $\vec{\beta}_\rho(1,2)$:

$$\vec{B}_\rho(1,2) = (\vec{p}_1 - \vec{p}_2)BW(1,2)$$
$$BW(1,2) = \frac{1}{M_{12}^2 - M_\rho^2 + iM_\rho\Gamma_\rho} \; , \tag{76}$$

where $\vec{p}_1$ and $\vec{p}_2$ are the three-momenta of particles 1 and 2 in the centre of mass of the ρ.

a) $\underline{I(\bar{p}p) = 0}$:

For $I(\bar{p}p) = 0$, only the $^3S_1$ initial state contributes (G-parity): the reaction can therefore be written:

$$1^- \rightarrow 1^- \;\; 0^-$$

$$\bar{p}p \quad \underbrace{\rho \quad \pi}_{1^-} \quad .$$

For the angular momentum between ρ and π, build the vector $\vec{p}_\rho - \vec{p}_3$ ($\vec{p}_\rho$ and $\vec{p}_3$ are now defined in the $\bar{p}p$ centre of mass) which, with $\vec{B}_\rho$, will be used to form a pseudovector:

$$\vec{B}_\rho \times (\vec{p}_\rho - \vec{p}_3)$$

$$A(1,2,3) = \vec{J} \cdot [(\vec{p}_1 - \vec{p}_2) \times (\vec{p}_\rho - \vec{p}_3)]BW(1,2) + \text{two other terms obtained by permutation,} \tag{77}$$

where $\vec{J}$ represents the initial state (1^-).

If the charges of the π mesons are associated to $\vec{p}_1$, $\vec{p}_2$, $\vec{p}_3$ in the following way:

$$\bar{p}p \rightarrow \pi_1^+ \pi_2^- \pi_3^0 \quad ,$$

we must single out the isospin part of the form $a_+ b_- c_0$. Only one such term is possible and the matrix element M must be written:

$$M(^3S_1 \to \rho\ \pi) = \vec{J} \cdot \{(\vec{p}_1 - \vec{p}_2) \times \vec{p}_3\ BW(1,2) + (\vec{p}_2 - \vec{p}_3) \times \vec{p}_1\ BW(2,3) + (\vec{p}_3 - \vec{p}_1) \times \vec{p}_2\ BW(3,1)\} \quad . \tag{78}$$

b) $\underline{I(\bar{p}p) = 1}$:

For $I(\bar{p}p) = 1$, only $^1S_0$ contributes. We have:

$$\begin{array}{ccc} 0^- & \to & 1^- \quad 0^- \\ \bar{p}p & & \underbrace{\rho \qquad \pi}_{1^-} \end{array} \quad .$$

One must form, with B_ρ and $(p_\rho - p_3)$, a scalar:

$$B(1,2) = \vec{B}_\rho(1,2) \cdot (\vec{p}_\rho - \vec{p}_3) \quad .$$

With the charge description given above, only $(\vec{a} \cdot \vec{b})\vec{c}$ will contribute and

$$M(^1S_0 \to \rho\ \pi) = (\vec{p}_2 - \vec{p}_3)p_1\ BW(2,3) + (\vec{p}_3 - \vec{p}_1)\vec{p}_2\ BW(3,1) \quad .$$

It is clear from this formula that $\rho^0 \to \pi_1^+ \pi_2^-$ will not be produced from the $^1S_0$ state. It is also clear that in this case the decay angular distribution of the ρ will be $\cos^2 \vartheta$, whereas it will be $\sin^2 \vartheta$ when produced by the $^3S_1$ state.

3.5 Dalitz-plot

We have seen that for three-body decay, we are left with two variables once the orientation of the system is fixed (for instance, by the direction of the normal to the decay plane, and the direction of one of the three particles in this plane).

The two remaining particles may be chosen in such a way that the transition probability writes:

$$d\Gamma = |M|^2\ dXdY.$$

This is effectively the case with the following choice of the variables X and Y:

- two of the three energies E_1, E_2, E_3
- two of the three kinetic energies T_1, T_2, T_3
- two of the three invariant mass squared:

$$M_{12}^2, \quad M_{23}^2, \quad M_{31}^2$$

(the energies and kinetic energies must be defined in the centre of mass of the decaying system).

The double dimension plot (E_1,E_2), or (T_1,T_2), or (M_{12}^2,M_{23}^2) will then exhibit the properties of $|M|^2$.

The technique described in Section 3.4 gives a key to the form of $|M|^2$ according to the spin-parity of the state, apart from unknown dynamical factors.

4. THE SO-CALLED "0^- WELL-ESTABLISHED NONET" AND THE EXPERIMENTAL SITUATION

4.1 Introduction

There is little doubt that the π, K, and η mesons form an octet of pseudoscalar particles. As already mentioned in the introduction, these particles are used as tools rather than studied for themselves. However, the branching ratios of the η^0 decay are still subject to some experimental work: we shall therefore recall the main features of the η meson.

The ninth pseudoscalar meson had a strange history: it was looked for without success for several years, and finally "discovered" (the η', or X^0, with a mass of 958 MeV) in 1964.

To be honest, its quantum numbers were not so well established; and, as we shall see from the experimental point of view, for the last two years it was not possible to decide rigorously between two possible assignments: $I^G J^P = 0^+0^-$ and 1^+1^+. But this X^0 meson fitted so well the "nonet hypothesis" that it was soon introduced in the "Tables" with the first assignment (0^+0^-).

However, in 1965, a charged meson was observed at practically the same mass as the X^0; it was called the δ^- and, of course, opened up the possibility that the X^0 was the neutral member of a multiplet of mesons, in complete disagreement with the "nonet hypothesis".

Soon after the discovery of this negatively charged meson, another experiment produced some evidence for the existence of a positively charged meson, with the mass and width observed for the δ^-, X^0: it was naturally called the δ^+.

We shall examine in some detail this $X^0 - \delta$ puzzle.

Although not directly connected with this difficulty, it seems interesting to proceed with a discussion of the properties of the E meson, which could very well be the tenth pseudoscalar meson, unless one has to abandon this assignment for the X^0: in this case, one would still end up with a "nonet" of pseudoscalar mesons.

4.2 The η meson

The η meson was first observed by Pevsner et al.[8] as a peak in the effective mass spectrum for the 3π combinations at

$$M(\pi^+\pi^-\pi^0) \sim 550 \text{ MeV}$$

in the reaction

$$\pi^+ + d \rightarrow p + p + \pi^+ + \pi^- + \pi^0 \quad \text{at } 1.23 \text{ GeV/c.} \tag{79}$$

No charged counterparts were found by Carmony et al.[9] in the (3π) combinations in the reactions:

$$\pi^\pm + p \rightarrow p + \pi^\pm + \pi^0 + \pi^0 \quad \text{at } 1.25 \text{ GeV/c.} \tag{80}$$

Since, for $I(\eta) = 1$, the cross-sections for production of η are related by the inequality

$$\sqrt{\sigma(\eta^+)} + \sqrt{\sigma(\eta^-)} \geq \sqrt{2\sigma(\eta^0)}$$

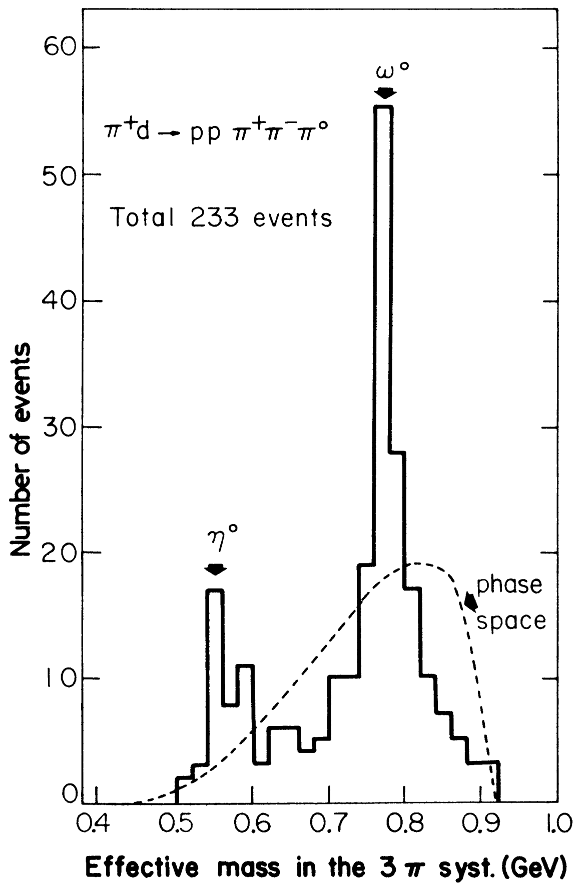


Fig. 1 Histogram of the effective mass of the three-pion system showing evidence for the η and the ω mesons[8].

for the reactions (79) and (80), it was concluded that $I(\eta) = 0$.

Another evidence for $I(\eta) = 0$ given by comparison of the production cross-sections in the two reactions:

$$K^- + p \rightarrow \Lambda + \pi^+ + \pi^- + \pi^0 \quad \text{[Bastien et al.}^{10)}] \qquad (81)$$

$$K^- + n \rightarrow \Lambda + \pi^+ + \pi^- + \pi^- \quad \text{[Prowse et al.}^{11)}] \qquad (82)$$

Since $(K^- + n)$ is a pure $I = 1$ state, and $I(\Lambda) = 0$, the cross-sections for production of η [if $I(\eta) = 1$] are related by the equality:

$$\sigma(\eta^-) = 2\sigma(\eta^0) \quad .$$

The experimental results for reactions (81) and (82) violating this equality, $I(\eta) = 0$ was unambiguously established.

If the η-meson decay into three pions is due to strong interactions, the isospin is conserved and one expects branching ratios in agreement with the following decomposition:

$$|3\pi,0\rangle = \frac{1}{\sqrt{6}} \Big\{ |\pi^+\pi^0\pi^-\rangle + |\pi^0\pi^-\pi^+\rangle + |\pi^-\pi^+\pi^0\rangle - |\pi^+\pi^-\pi^0\rangle - |\pi^0\pi^+\pi^-\rangle - |\pi^-\pi^0\pi^+\rangle \Big\} \quad .$$

In other words, two of the three pions are always charged mesons, and the η decay does not lead to a complete neutral system ($\eta \not\rightarrow 3\pi^0$). Indeed, this is in complete disagreement with the experimental situation: the comparison of the cross-sections for η production between reaction (81′) and (83):

$$K^- + p \rightarrow \Lambda + \eta_{\rightarrow \pi^+\pi^-\pi^0} \tag{81′}$$

$$K^- + p \rightarrow \Lambda + \eta_{\rightarrow \text{all neutrals}} \tag{83}$$

shows that the complete neutral decay mode is more important than the charged one ($\eta \rightarrow \pi^+\pi^-\pi^0$):

$$\frac{\Gamma(\pi^+\pi^-\pi^0)}{\Gamma(\text{all neutrals})} = 30\% \quad .$$

The conclusion is that isospin is not conserved in the η^0 decay: electromagnetic interactions, and not strong interactions, have therefore to be considered for the observed decay modes.

This conclusion is confirmed by the study of the Dalitz plot for $\eta \rightarrow \pi^+\pi^-\pi^0$.

Since for I = 0, the isospin part of the matrix element built with three isovectors (three pions) is completely antisymmetric in the exchange of two particles (see Section 3), the spin-parity part of the complete matrix element must also be completely antisymmetric in the exchange of two particles: this implies that this matrix element must be zero at certain points of the Dalitz plot:

for example: if $J^P(\eta) = 0^-$, $M \sim (E_1 - E_2)\ (E_2 - E_3)\ (E_3 - E_1)$

where E_1, E_2, E_3 are the energies of the three pions. The population of the Dalitz plot must then vanish on the three lines defined by

$$E_1 = E_2, \quad E_2 = E_3, \quad E_3 = E_1 \quad .$$

If $J^P(\eta) = 1^-$, the matrix element must be a pseudovector:

$$M \sim \vec{p}_1 \times \vec{p}_2 \quad ,$$

$\vec{p}_1$ and $\vec{p}_2$ being the momenta of the pions 1 and 2. This matrix element vanishes when the pions are colinear, i.e. on the edge of the Dalitz plot.

Indeed, the experimental distribution of events on the $\eta^0 \rightarrow \pi^+\pi^-\pi^0$ Dalitz plot does not agree with these predictions, and can only be explained if the antisymmetric properties of the matrix element are abandoned, that is if one assumes that the three-pion final state has a contribution from I = 1.

The simplest way to explain the experimental distribution of the points on the Dalitz plot is to assume $J^P = 0^-$ for the η meson.

Independent evidence for the spin-parity of the η meson comes from the observation of the two-photon decay mode:

$$\eta^0 \rightarrow 2\gamma \quad \text{[Chrétien et al.[12]]} \quad .$$

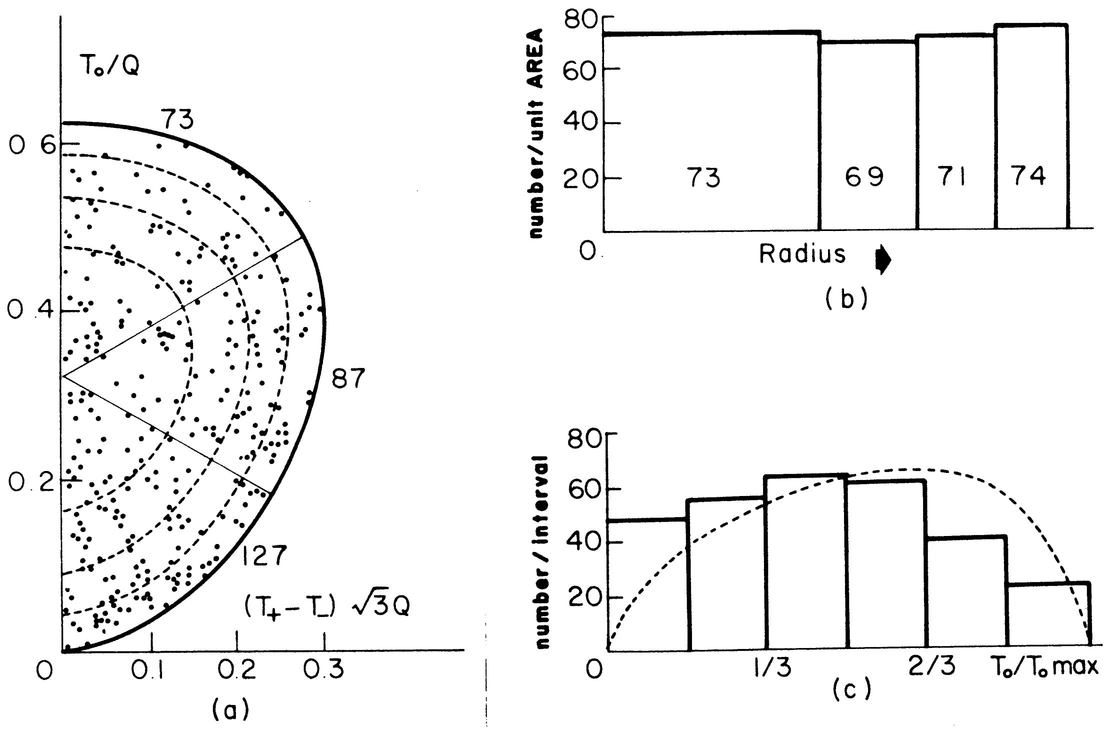


Fig. 2 The Dalitz plot and projections for $\eta^0 \to \pi^+\pi^-\pi^0$.

a) shows the distribution of the points on a "normalized" Dalitz plot (T_o, T_+, T_- are for the kinetic energy of the π^0, π^+, π^-, respectively; Q is equal to the total kinetic energy available in the η^0 centre of mass). Only half of the kinematically allowed region is shown, since by C conservation, the Dalitz plot must be symmetric under the exchange of the π^+ and π^-.

b) shows the radial distribution. It does not vanish at the boundary as it should do if the η^0 had the spin parity 1^-.

c) shows the projection of the points on the π^0 axis. It is clearly not equal to zero, as it should be if the η^0 had the spin parity 1^+. The dotted line corresponds to a uniform population. The observed deviation from uniformity is not inconsistent with $J^P = 0^-$ for the η^0, but it means that final-state interactions take place among the pions[13].

This observation rules out $J(\eta) = 1$; it confirms $J^P = 0^-$ and shows that $C(\eta) = +1$; therefore $G(\eta) = C(-1)^I = +1$.

As expected, the decay $\eta \to \pi^+\pi^-\pi^0$ violates the G-parity.

With these quantum numbers ($I^G J^P = 0^+ 0^-$), the η meson could decay into four pions, but this decay is severely limited by the available phase space (indeed just possible for $4\pi^0$ decay).

The branching ratios of the η into its various decay modes are not yet well understood; since both decays, $\eta \to \pi^+\pi^-\pi^0$ and $\eta \to \gamma\gamma$, are electromagnetic, they should be proportional

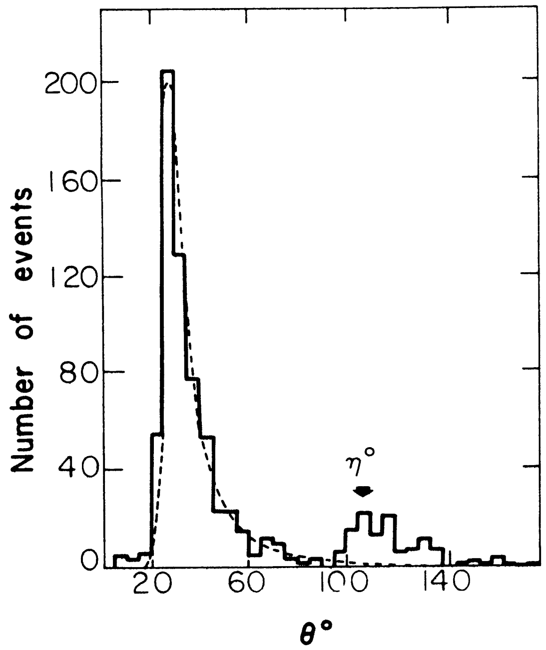


Fig. 3 Opening angle distribution of 2γ decays. The curve corresponds to the expected distribution for the decay $\pi^0 \to 2\gamma$. The excess of events around $\vartheta = 120°$ corresponds to the decay $\eta^0 \to 2\gamma$ [12].

to α^2 ($\alpha = 1/137$, fine structure constant) and phase space should favour $\eta \to \gamma\gamma$ with respect to $\eta \to \pi^+\pi^-\pi^0$ by an order of magnitude. Indeed, these two decays have comparable rates:

$$\Gamma(\eta \to \gamma\gamma) \ / \ \Gamma(\eta \to \pi^+\pi^-\pi^0) = 1.4 \pm 0.2 \quad .$$

Several explanations have been put forward in order to understand this anomaly. $\eta \to \gamma\gamma$ is forbidden by a new conservation law [the A-parity, see Bronzan and Low[14]]. perhaps by a resonance in the pion-pion system [Brown and Singer[15]].

The A-parity assumes $A_\pi = A_\eta = -1$, whereas $A_\gamma = +1$. When it is violated, it is assumed that a reduction factor ϵ of the order of 1% is introduced, If, in addition, one takes into account a phase space factor ρ (of the order of 1% for the three-body decay modes of the η with respect to the two-body decay mode $\eta \to \gamma\gamma$), we are able to reproduce, roughly, the observed branching ratios:

$\eta \to \gamma\gamma$	31%	ϵ
$\eta \to \pi^0\gamma\gamma$	20.5%	ρ
$\eta \to 3\pi^0$	21%	ρ
$\eta \to \pi^+\pi^-\pi^0$	22.4%	ρ
$\eta \to \pi^+\pi^-\gamma$	4.6%	$\rho \cdot \epsilon$.

All the decay modes are comparable (implying $\rho \sim \epsilon$) except that of the $\pi^+\pi^-\gamma$, which is expected to be smaller since both phase space and A-parity play against it.

On the other hand, with the final-state interaction hypothesis, one does not understand how the decay $\eta \to \pi^0\gamma\gamma$ is enhanced with respect to $\eta \to \gamma\gamma$. One is also in trouble with the observation:

$$\Gamma(\eta \to 3\pi^0) \sim \Gamma(\eta \to \pi^+\pi^-\pi^0)$$

since this ratio should be 1/2 with final-state interaction.

4.3 The $X^0(\eta') - \delta^{\pm}$ puzzle

The first evidence for a neutral unstable meson with a mass of ~ 950 MeV was reported by Goldberg et al.[16], Kalbfleisch et al.[17], and Dauber et al.[18]. This evidence came from the study of K^-p interactions with production of a Λ^0:

$$K^- + p \to \Lambda^0 + X^0 \; . \tag{84}$$

Two modes of decay were observed for the X^0:

$$X^0 \to \eta^0\pi^+\pi^- \quad 75\% \tag{85}$$

$$X^0 \to \gamma\pi^+\pi^- \quad 25\% \; , \tag{86}$$

On the other hand, the two-pion, three-pion, and four-pion decay modes have been looked for without success:

$$\begin{aligned} X^0 &\to 2\pi &< 7\% \\ X^0 &\to 3\pi &< 7\% \\ X^0 &\to 4\pi &< 1\% \; . \end{aligned} \tag{87}$$

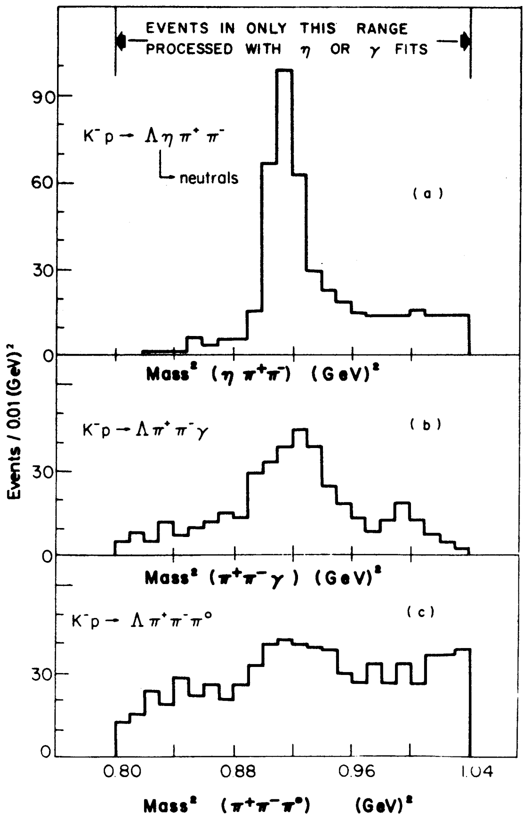


Fig. 4 Evidence for the $\pi^+\pi^-\eta$ and $\pi^+\pi^-\gamma$ decay modes of the X^0 [19].

a) $K^-p \to \Lambda\eta\pi^+\pi^-$: the histogram shows the effective mass squared distribution of the $\eta\pi^+\pi^-$ system. The peak at $M^2 = 0.92$ GeV^2 corresponds to the X^0.

b) $K^-p \to \Lambda\pi^+\pi^-\gamma$: the histogram shows the effective mass squared distribution of the $\pi^+\pi^-\gamma$ system. There is an accumulation of events around $M^2 = 0.92$ GeV^2.

c) $K^-p \to \Lambda\pi^+\pi^-\pi^0$: the accumulation tends to disappear around $M^2 = 0.92$ GeV^2.

These observations lead to the conclusion that the decay mode (85) is very probably due to strong interactions; therefore

$$G(X^0) = +1 \quad .$$

The argument is the following: if the decay mode (85) was due to electromagnetic interactions, then $G(X^0)$ would be -1, and the decay mode (87) should be a strong one, since it is allowed for all possible J^P assignments for the X^0 when $G(X^0) = -1$ (with the exception of $J^P = 0^+$ which forbids the $\eta\pi\pi$ decay as well.)

The A-parity cannot be invoked since it has an equivalent effect on $\pi\pi\eta$ and $\pi\pi\pi$ systems.

One would therefore expect a much more abundant 3π decay than $\eta\pi\pi$ (by at least a factor of 10^4) in complete disagreement with the experimental situation.

Therefore, the decay mode (85): $X^0 \to \eta^0\pi^+\pi^-$, is certainly a strong one, and $G(X^0) = +1$.

The decay matrix elements for such a decay are shown in Table 4, for the simplest I^GJ^P assignments:

Table 4

I^GJ^P	Simplest L, ℓ	$M(\vec{p},\vec{q})$	Density of the Dalitz plot
0^+0^-	0,0	1	1
1^+	0,1	$\vec{p}$	p^2
1^-	2,2	$(\vec{p}\cdot\vec{q})\ (\vec{p}\times\vec{q})$	$p^4q^4\cos^2\vartheta\sin^2\vartheta$
1^+0^-	1,1	$\vec{p}\cdot\vec{q}$	$p^2q^2\cos^2\vartheta$
1^+	1,0	$\vec{q}$	q^2
1^-	1,1	$\vec{p}\times\vec{q}$	$p^2q^2\sin^2\vartheta$

where $\vec{p}$ and $\vec{q}$ denote the momentum of the π mesons in the $(\pi\pi)$ centre of mass, and the momentum of the η^0 in the X^0 centre of mass; ϑ is the angle between $\vec{p}$ and $\vec{q}$, measured in the $(\pi\pi)$ centre of mass. L is the orbital angular momentum of the $(\pi\pi)$ system, ℓ the relative orbital angular momentum of the η^0.

Unfortunately, the experimental data does not allow one to disentangle, unambiguously, some of these assignments: 0^+0^- and 1^+1^+ are both possible. Moreover, possible final-state interactions may increase the difficulties of this analysis: if a 0^+0^+ dipion meson (the "σ" meson) is present in nature, it may affect the distribution of events in the three-body decay $X^0 \to \eta\pi^+\pi^-$, and the decay matrix elements mentioned above have to be modified to be compared with the experimental data.

In view of these difficulties, it is interesting to see how the decay mode (86) $X^0 \to \pi^+\pi^-\gamma$ can be used to help in the determination of the spin parity of the X^0.

One limits oneself to the lowest order electromagnetic transitions, allowed by conservation of spin and parity.

Let us first examine:

$$I^GJ^P\ (X^0) = 0^+0^-$$

$$X^0 \to \pi^+\pi^-(1^+1^-) + \gamma \quad .$$

The decay matrix element squared will be proportional to $(\vec{p} \times \vec{q})^2$, where $\vec{p}$ is the momentum of the π mesons in the dipion centre of mass and $\vec{q}$ the momentum of the photon in the over-all centre of mass: then quantum numbers for the dipion will favour the production of the ρ^0 meson, and its angular distribution will be $\sim \sin^2 \vartheta$, where ϑ is the angle between $\vec{p}$ and $\vec{q}$, measured in the dipion centre of mass.

These two properties (accumulation around the mass of the ρ^0 meson and $\sin^2 \vartheta$ decay angular distribution) are in agreement with the experimental data.

If

$$I^G J^P (X^0) = 1^+ 1^+ \quad ,$$

$$X^0 \to \pi^+\pi^-(0^+0^-) + \gamma \quad ,$$

and the decay matrix element would be proportional to q^2, in contrast with the experimental data.

Therefore, if the decay $\pi^+\pi^-\gamma$ is attributed to the same particle as is the $\eta\pi^+\pi^-$ decay discussed above,

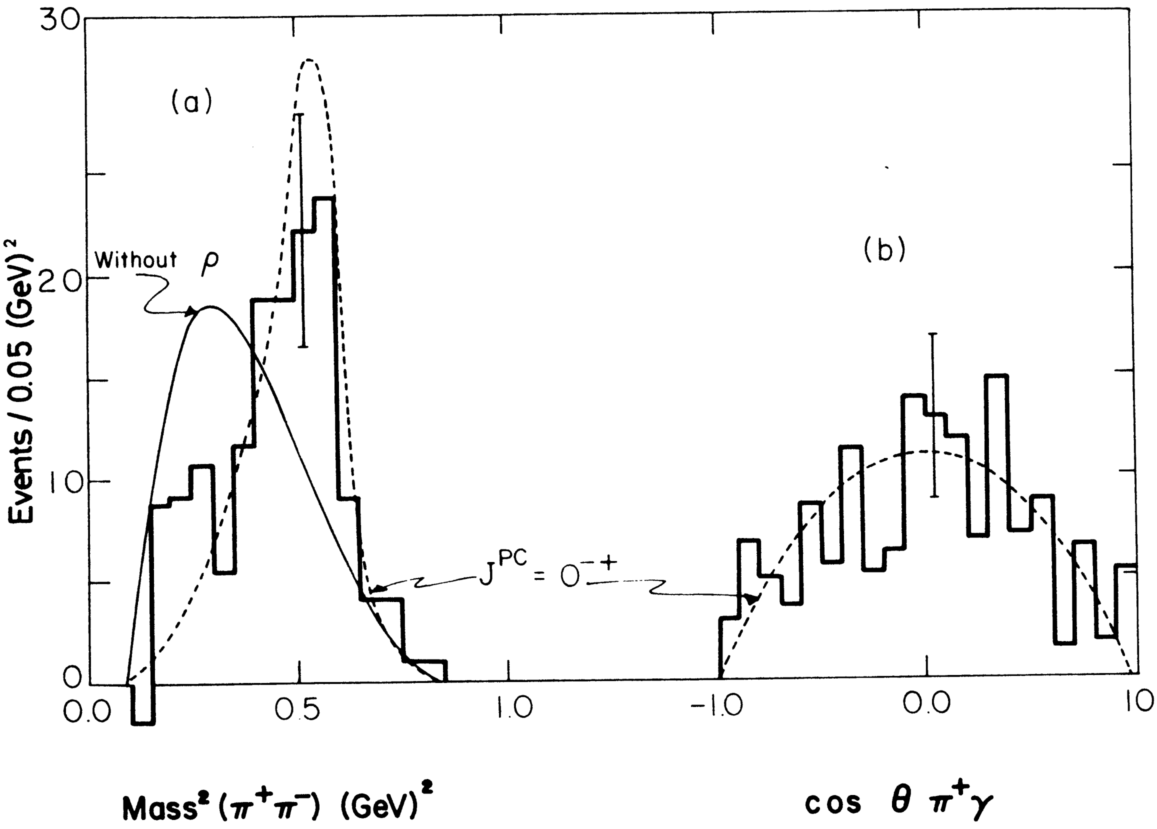


Fig. 5 $X^0 \to \pi^+\pi^-\gamma$ [19].

a) Effective mass squared distribution of the $(\pi^+\pi^-)$ system, showing the accumulation towards the ρ mass.

b) Decay angular distribution of the $(\pi^+\pi^-)$ system, showing the compatibility with $\sin^2 \Theta$, in agreement with spin parity 0^- for the X^0.

$$G(X^0) = +1$$
$$C(X^0) = C(\rho^0) \cdot C(\gamma) = +1 \quad ,$$

thus $$I(X^0) = 0 \quad ,$$

and the assignment 1^+1^+ must be rejected.

A more direct determination of the isospin of the X^0 is obtained by the comparison of the production cross-sections for the reactions:

$$K^- + p \rightarrow \Lambda^0 + X^0 \tag{84}$$

$$K^- + n \rightarrow \Lambda^0 + X^- \quad . \tag{88}$$

If $I(X^0) = 0$, of course reaction (88) will not be observed; but if $I(X^0) = 1$, σ (84)/σ (88) = ½.

Two experiments have provided, independently, the same result: whereas 19 ± 6 events were expected to be observed for reaction (88), none was observed; I = +1 is therefore very unlikely for the X^0 meson[20,21].

However, two missing-mass spectrometer experiments have led to the observation of a narrow charged meson resonance at a mass very close to the X mass; Kienzle et al.[22] studied the reaction

$$\pi^- + p \rightarrow p + (MM)^- \quad .$$

They found a resonance, called δ^- at M = 962 ± 5 MeV with a width $\Gamma \leq 5$ MeV.

Oostens et al.[23] studied the reaction

$$p + p \rightarrow d + (MM)^+$$

and again found a narrow resonance at a mass M = 966 ± 6 MeV with a width $\Gamma < 5$ MeV. This last observation implies $I(\delta^+) = 1$.

If all these experimental results are confirmed, one has to ask the question: are the X^0 and $\delta^\pm$ members of the same multiplet, or are we in the presence of two distinct multiplets (a singlet X^0, and a triplet δ)? For the moment, it may be better to reserve our judgement, although the experimental evidence gives strong support to the existence of an isoscalar meson, often referred to as the $\eta'(958)$ meson, with $J^P = 0^-$. For a more detailed discussion of the δ meson (I = 1), see also Section 7.1 (I = 1 $K\bar{K}$ structure near threshold).

4.4 The E^0 meson

We introduce here the discussion on the E meson, since the experimental results strongly support the assignment $J^P = 0^-$.

The first evidence for a neutral meson with a mass of 1420 MeV and a width $\Gamma \sim 70$ MeV decaying into a $K\bar{K}\pi$ system has been reported by Armenteros et al.[24].

The observation was made on $\bar{p}p$ annihilations at rest:

$$\bar{p}p \rightarrow K_1^0 K^\pm \pi^\mp \pi^+ \pi^- \quad . \tag{89}$$

The E meson has also been observed in $\bar{p}p$ annihilations in flight: at 3.6 GeV [French et al.[25]]; and at 1.2 GeV/c [D'Andlau et al.[26]]. Hess et al.[27] have reported its presence in π^-p interactions.

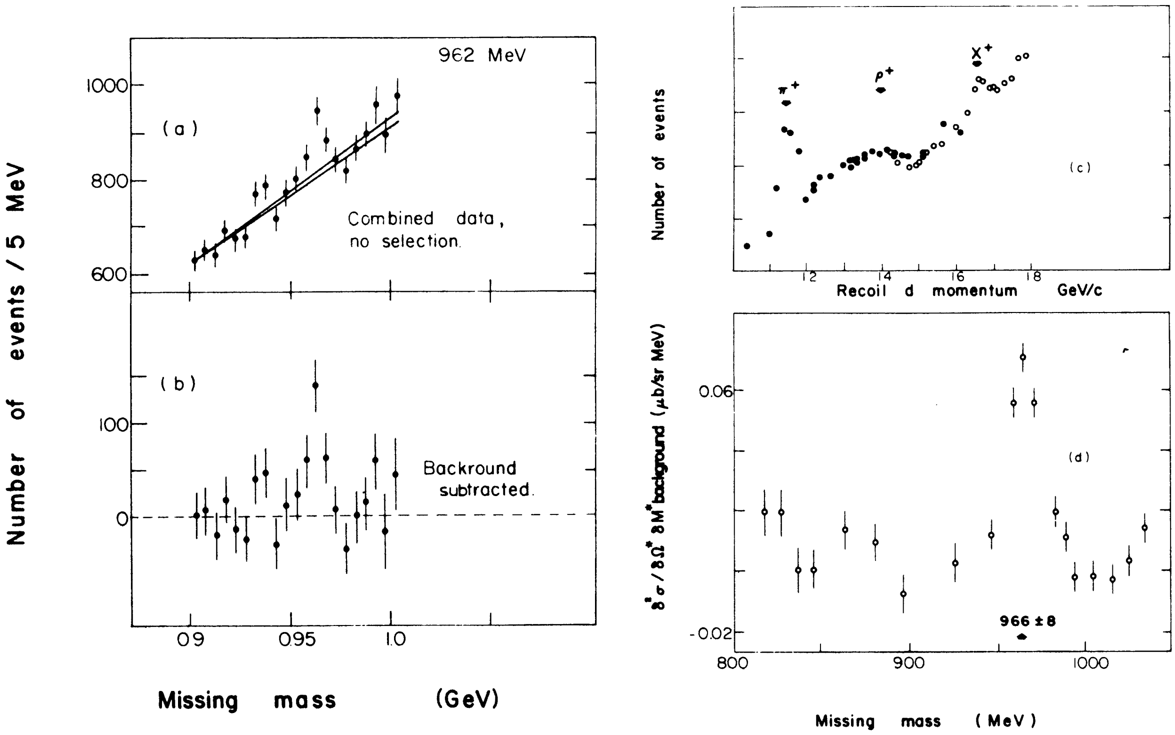


Fig. 6 Evidence for the $\delta^\pm$ meson.

a) Missing-mass spectrometer experiment[22]. The histogram shows the missing-mass distribution for the combined data, without subtraction.

b) The same data, after subtraction of a uniform background. The peak at 962 MeV has been called δ^-.

c) Missing-mass spectrometer experiment[23]. The histogram shows the momentum distribution of the deuteron for the reaction $p + p \to d + X$.

d) The same data, after subtraction of a uniform background in the region of the δ meson. There is a peak at 966 MeV, identified with the positive counterpart of the δ^-.

A detailed analysis of the quantum numbers of the E meson has been recently given by Baillon et al.[28]: it results from the study of 1.4×10^6 $\bar{p}p$ annihilations, which provided the following data:

$$\bar{p}p \to K^0_1 K^\pm \pi^\mp \pi^+ \pi^- \qquad 600 \text{ events} \tag{89}$$

$$\bar{p}p \to K^0_1 K^\pm \pi^\mp (\pi^0 \pi^0) \qquad 273 \text{ events} \tag{90}$$

$$\bar{p}p \to K^0_1 K^0_1 \pi^+ \pi^- \pi^0 \qquad 657 \text{ events} \tag{91}$$

$$\bar{p}p \to K^0_1 (K^0 \pi^0) \pi^+ \pi^- \qquad 757 \text{ events} \tag{92}$$

$$\bar{p}p \to K^+ K^- \pi^+ \pi^- \pi^0 \qquad 740 \text{ events} \tag{93}$$

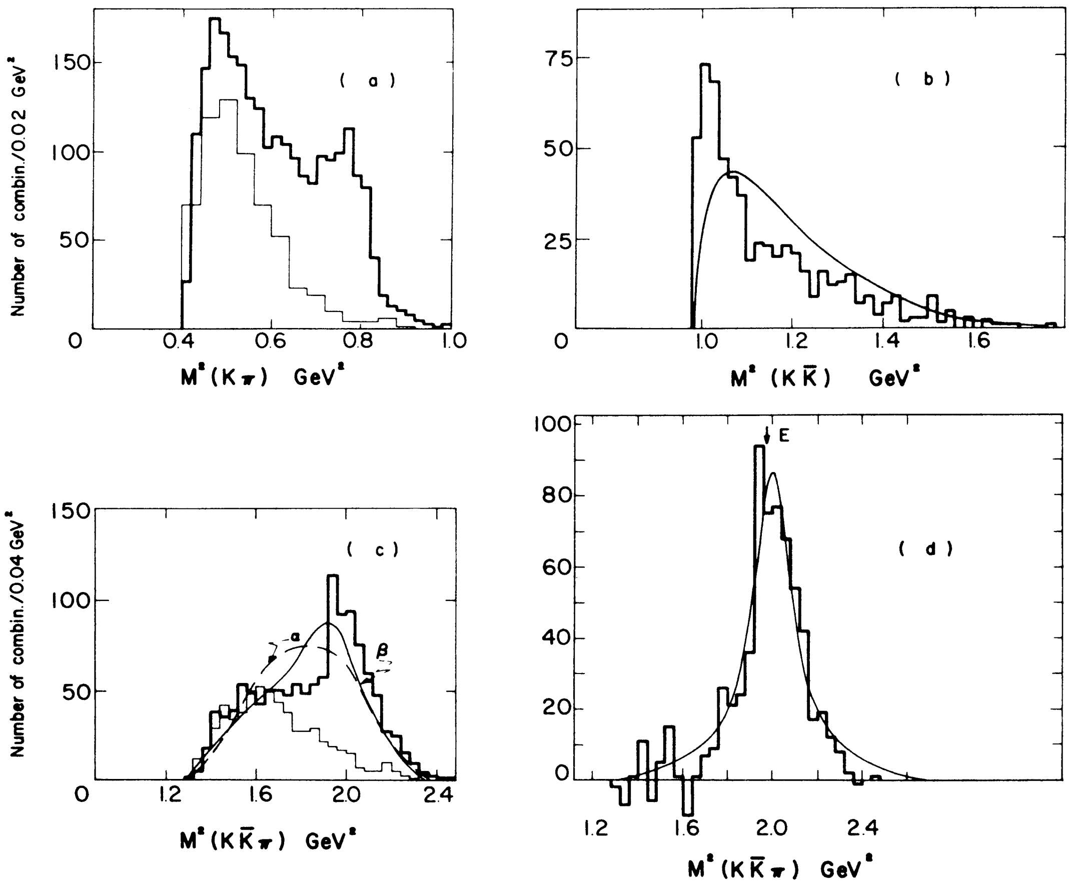


Fig. 7 Evidence for the E meson in $\bar{p}p \to K_1^0K^{\pm}\pi^{\mp}\pi^+\pi^-$ [28].

a) $(K\pi)$ effective mass squared spectra. The dark line gives the distribution of $(K\pi)_{I=1/2}$ (four combinations per event), whereas the grey line gives the distribution of $(K\pi)_{I=3/2}$ (two combinations per event). The production of $K^*(891)$ is clearly seen on the $(K\pi)_{I=1/2}$ distribution.

b) $(K\bar{K})$ effective mass spectrum. The solid line corresponds to the phase-space distribution. One observes an accumulation of events near threshold.

c) $(K\bar{K}\pi)$ effective mass spectra. The dark line gives the distribution of $(K\bar{K}\pi)_{Q=0}$ (two combinations per event), whereas the grey line gives the distribution of $(K\bar{K}\pi)_{Q=2}$ (one combination per event). Curves α and β correspond, respectively, to phase-space and 100% K^* production with constructive interferences effects. The enhancement seen in the $Q = 0$ distribution has been taken as an evidence for the E meson.

The reactions (90) and (92) do not give rise to a fit since there are two missing particles; they have been selected from the events which do not give a four-body fit. Since no six-body annihilations have been observed at rest, they can safely be attributed to the five-body channels.

The E meson could also be produced, in principle, in four-body annihilations:

$$\bar{p}p \to K_1^0K^{\pm}\pi^{\mp}\pi^0 \quad . \tag{94}$$

These annihilations have been analysed; no E^0 production has been observed. We shall see later that one expects this absence of $\bar{p}p \to E^0\pi^0$ (by spin-parity arguments).

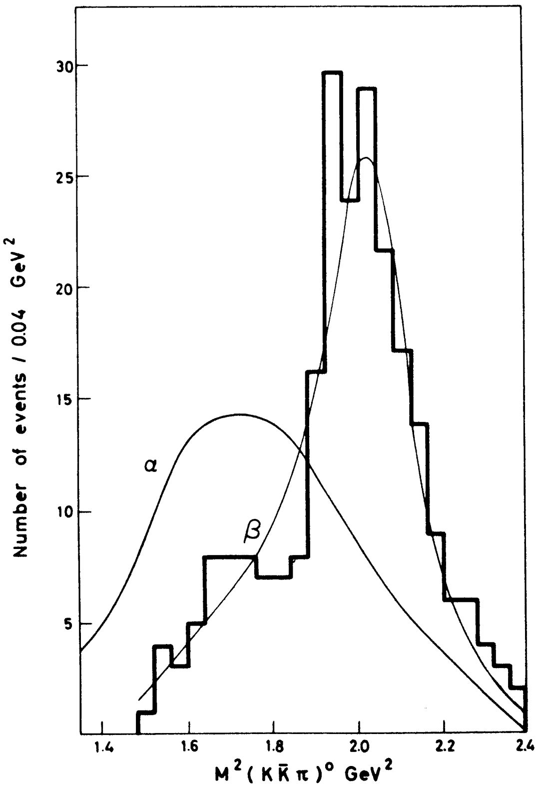


Fig. 8 Evidence for the E meson in $\bar{p}p \rightarrow K_1^0 K^{\pm}\pi^{\mp}\pi^0\pi^0$ [28] $(K\bar{K}\pi)^0$ effective mass spectrum. Curve α corresponds to phase space. Curve β is obtained for a Breit-Wigner distribution with the mass and width of the E meson (M = 1425 MeV, Γ = 80 MeV).

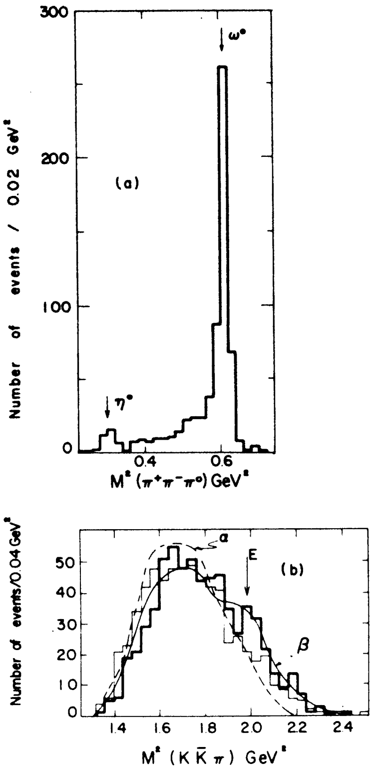


Fig. 9 Evidence for the E meson in $\bar{p}p \rightarrow K_1^0 K_1^0 \pi^+\pi^-\pi^0$ [28].

a) $(\pi^+\pi^-\pi^0)$ effective mass spectrum. This distribution shows that this five-body annihilation is dominated by the production of ω^0 (70%) and, to a lesser extent, by the η^0.

b) $(K\bar{K}\pi)$ effective mass spectra. The dark line gives the distribution of $(K\bar{K}\pi)_{Q=0}$ (one combination per event), whereas the grey line gives the distribution of $(K\bar{K}\pi)_{Q=\pm 1}$ (two combinations per event).

Curve α represents the effect of the ω^0 production on the $(K\bar{K}\pi)$ system.

Curve β is obtained when adding to the ω^0 production (70%), 18% of background (phase space) and 12% of E^0 production. It corresponds to the best fit and demonstrates the existence of the decay $E^0 \rightarrow K_1^0 K_1^0 \pi^0$, i.e. C(E) = +1.

The best evidence for the $E^0 \to K_1^0 K^{\pm}\pi^{\mp}$ is provided by reaction (89), in particular when the double-charged $(K_1^0 K^{\pm}\pi^{\pm})$ mass spectrum is subtracted from the neutral one $(K_1^0 K^{\pm}\pi^{\mp})$ (for which there are two combinations per event).

However, the $(K\pi)$ mass spectrum for these events shows an abundant production of $K^*(891)$, and one may ask the question : is the $(K\bar{K}\pi)^0$ peak at 1420 MeV a reflection of the K^* meson? To answer this question, one computes the effect of the K^* meson on the $(K\bar{K}\pi)$ spectrum, taking into account that four K^* amplitudes may be present:

$$K^0 K^- \pi^+ \pi^+ \pi^-$$
$$1 \; 2 \; 3 \; 4 \; 5$$
$$M = (A_{13} + A_{14}) \pm (A_{23} + A_{24}) \quad . \tag{95}$$

Equation (95) is symmetrized under the exchange of the two identical pions. The remaining phase is not known and several hypotheses must be tried.

For the amplitudes A, one takes the usual Breit-Wigner form:

$$A_{ij} = \frac{1}{m_{ij}^2 - m^{*2} + i m^* \Gamma}$$

where m^* is the mass of $K^*(891)$;

m_{ij} is the effective mass of particles i and j; and,

Γ is the width of $K^*(891)$.

The results of this calculation show that the $K^*(891)$ (even in the extreme assumption of 100% production with complete constructive interferences effects) do not reproduce the 1420 MeV$(K\bar{K}\pi)$ peak.

Another two-body effect is apparent in reaction (89): the $K_1^0 K^{\pm}$ mass spectrum is sharply peaked near threshold.

If one assumes that this is a genuine effect, one may ask again if the 1420 MeV $(K\bar{K}\pi)$ peak is the reflection of such a $(K\bar{K})$ accumulation near threshold. The answer is again no, as can be easily guessed if one observes that such an effect should alter the double-charged $(K\bar{K}\pi)$ system in the same way as the neutral one.

We conclude from this analysis that the peak observed in the neutral $(K\bar{K}\pi)$ system is due to the presence of a new resonance, hereafter called the E meson:

$$\bar{p}p \to E^0 \pi^+ \pi^- \quad . \tag{96}$$

Indeed, one finds that the entire reaction (89) is compatible with reaction (96), i.e. the "background" is less than 10%.

The same conclusions are reached for the analysis of reaction (90), which is dominated by the production of the E^0 meson. The "background" is estimated to be less than 25%:

$$\bar{p}p \to E^0 \pi^0 \pi^0 \quad . \tag{97}$$

From reactions (96) and (97), the mass and width of the E are found to be:

$$M = 1425 \pm 7 \text{ MeV}, \qquad \Gamma = 80 \pm 10 \text{ MeV} \quad .$$

4.5 Charge conjugation and isospin of the E

The absence of double-charged $(K\bar{K}\pi)$ enhancement in reaction (89) is taken as evidence against I(E) = 2, since, assuming I(E) = 2 and non-zero transition probability for the process

$$\bar{p}p \to E^{++}\pi^-\pi^- \quad ,$$

there is only one decay amplitude for the E, therefore no possible cancellation due to interference effects.

We thus limit ourselves to the possible assignments

$$C(E) = \pm 1 \qquad I(E) = 0,1 \quad .$$

Let us consider the isospin decomposition of the $(K\bar{K}\pi)$ systems. We need two amplitudes A_1 and A_0, referring to $I(K\bar{K}) = 1$ and $I(K\bar{K}) = 0$, respectively.

In general, this decomposition will lead to terms of the form

$$(K^0K^{\pm}\pi^{\mp})$$
$$(K_1^0K_1^0\pi^0)$$
$$(K_1^0K_2^0\pi^0)$$
$$(K^+K^-\pi^0)$$

for the neutral $(K\bar{K}\pi)$ system, and to terms of the form:

$$(K^0K^{\pm}\pi^0)$$
$$(K_1^0K_1^0\pi^{\pm})$$
$$(K_1^0K_2^0\pi^{\pm})$$
$$(K^+K^-\pi^{\pm})$$

for the charged $(K\bar{K}\pi)$ system.

However, some of these terms do not appear when the charge conjugation C is fixed for the E:

$$C(E) = +1 \quad \text{does not lead to } (K_1^0K_2^0\pi^0)$$
$$C(E) = -1 \quad \text{does not lead to } (K_1^0K_1^0\pi^0) \quad .$$

We have also to consider two different transition rates, M and N, depending on the charge of the E meson:

$$\bar{p}p \to E^0\pi^+\pi^- \; : \; M$$
$$\bar{p}p \to E^{\pm}\pi^{\mp}\pi^0 \; : \; N \quad .$$

C(E) = -1, both for I(E) and I(E) = 1, gives bad predictions when compared to the experimental results:

Table 5

	Predictions for C = -1 I = 0	Predictions for C = -1 I = 1	Observations
$K_1^0K^{\pm}\pi^{\mp}$	600*)	600*)	600
$K_1^0K_1^0\pi^0$	0	0	83
$K_1^0K_2^0\pi^0$	300	20*)	< 20
$K^+K^-\pi^0$	497	33	> 150

The predictions for C(E) = -1 are bad for two reasons:

i) One should not observe the decay $E^0 \to K_1^0K_1^0\pi^0$; but the study of reaction (91):

$$\bar{p}p \to K_1^0K_1^0\pi^+\pi^-\pi^0$$

shows that 12% of this channel should be attributed to $\bar{p}p \to E^0\pi^+\pi^-$, i.e. 83 ± 21 events, when C(E) = +1 implies that 90 ± 25 E^0 will be observed with this decay mode [for I(E) = 0].

[The remaining events of reaction (91) come essentially from the channel $\bar{p}p \to K_1^0K_1^0\omega^0$.]

ii) One should observe the decay mode $E^0 \to K_1^0K_2^0\pi^0$: if I(E) = 0, the prediction is in clear disagreement with the observation, since no event can be attributed to this decay mode. An upper limit (< 20) has been obtained by the comparison of the $(K\bar{K}\pi)^0$ spectra for reactions (91) and (92).

For I(E) = 1, however, this result cannot be taken as a prediction, since the two amplitudes A_0 and A_1 participate in the decay. The upper limit (20 events) may be then taken as a normalization factor to predict the decay rate $E^0 \to K^+K^-\pi^0$.

If the predictions for $E^0 \to K^+K^-\pi^0$, when I(E) = 0, are compatible with the observation, they are much too small for I(E) = 1. For this channel we have only a lower limit, since the events must be found in the "four-prong" annihilations which are dominated by the pion channels: the scanning and analysis of a limited sample of such events has provided a lower limit of 150 $E^0 \to K^+K^-\pi^0$, much more than the expected number for C = -1, I = 1.

We conclude that C(E) = +1.

The isospin I(E) is less easy to determine.

I(E) = 0 assignment gives very good agreement with the observations:

*) Taken as a normalization.

Table 6

C = +1, I = 0		
	Predictions	Observations
$K_1^0K^{\pm}\pi^{\mp}$	600*)	600
$K_1^0K_1^0\pi^0$	90	83
$K_1^0K_2^0\pi^0$	0	< 20
$K^+K^-\pi^0$	497	> 150

*) Taken as a normalization.

However, to exclude I(E) = 1, we have to consider the predictions for the decay of the charged E meson: from the experimental point of view, only two modes are observable:

$$E^{\pm} \to K_1^0K_1^0\pi^{\pm}\ ,\quad E^{\pm} \to K^+K^-\pi^{\pm}\ .$$

No event can be attributed to the E in these channels, which means that I(E) = 1 is excluded, unless the production of the $E^{\pm}$ is cancelled by some mechanism. We have therefore to examine the conditions which may lead to N = 0.

We have: C(E) = +1, and assume I(E) = 1,

thus

$$G(E) = -1$$
$$G(E\pi\pi) = -1$$
$$G(\bar{p}p) = -1\ .$$

This relation is satisfied for:

$$C(\bar{p}p) = +1\ ,\ I(\bar{p}p) = 1$$

or

$$C(\bar{p}p) = -1\ ,\ I(\bar{p}p) = 0\ .$$

We have therefore to consider four production amplitudes:

$$\left.\begin{aligned} \beta_{12} &= \langle \bar{p}p\ I = 1,\ C = 1 \mid E\pi\pi\ I(\pi\pi) = 2\rangle \\ \beta_{11} &= \langle \bar{p}p\ I = 1,\ C = 1 \mid E\pi\pi\ I(\pi\pi) = 1\rangle \\ \beta_{10} &= \langle \bar{p}p\ I = 1,\ C = 1 \mid E\pi\pi\ I(\pi\pi) = 0\rangle \end{aligned}\right\}\ {}^1S_0$$
$$\beta_{01} = \langle \bar{p}p\ I = 0,\ C = -1 \mid E\pi\pi\ I(\pi\pi) = 1\rangle \ \Big\}\ {}^3S_1\ .$$

In terms of the charged modes, we get:

$$\bar{p}p \to E^{\pm}\pi^{\mp}\pi^0 \qquad N = \frac{3}{5}\,|\beta_{12}|^2 + |\beta_{11}|^2 + \frac{2}{3}\,|\beta_{01}|^2$$

$$\bar{p}p \to E^0\pi^+\pi^- \qquad M_1 = \frac{2}{3}\left[\left|\beta_{10} - \frac{1}{\sqrt{5}}\beta_{12}\right|^2 + \frac{1}{2}\,|\beta_{01}|^2\right]$$

$$\bar{p}p \to E^0\pi^0\pi^0 \qquad M_2 = \left|\sqrt{\frac{4}{15}}\,\beta_{12} + \frac{1}{\sqrt{3}}\,\beta_{10}\right|^2\ .$$

N will be zero only if $\beta_{12} = \beta_{11} = \beta_{01}$.

This condition seems to be very unlikely; therefore I(E) = 1 is unlikely and we conclude:

$$C(E) = +1 \ , \ I(E) = 0 \ .$$

4.6 Spin parity of the E meson - E decay

Since $G(E) = +1$, $G(K\bar{K}) = -1$. But $I(K\bar{K}) = 1$ (at least for the mode $K_1^0 K^\pm \pi^\mp$ which we shall consider now). Thus the orbital angular momentum $L(K\bar{K})$ must be even and $J^P(K\bar{K}) = 0^+, 2^+$.

The $K\bar{K}$ mass spectrum associated to the E decay shows a remarkable accumulation towards $K\bar{K}$ threshold: this fact favours the assignment $J^P(K\bar{K}) = 0^+$ rather than 2^+.

We shall therefore assume $J^P(K\bar{K}) = 0^+$; this leads to the possible assignments for the E:

$$J^P = 0^-, 1^+, 2^- \ .$$

Another observation leads to the introduction of K* interference effects in the decay of the E: the decay angular distribution of the $K\bar{K}$ system is not flat, as would be expected for $J^P(K\bar{K}) = 0^+$; the non-uniformity can be easily explained by the presence of K* interference effects (there are two K* amplitudes for the $K\bar{K}\pi$ system).

We therefore assume two decay modes for the E:

$$E \to (K\bar{K})\pi \qquad [\text{with } J^P(K\bar{K}) = 0^+]$$

$$E \to K^*\bar{K}(\bar{K}^*K) \ .$$

The decay matrix elements are thus written:

for:

$$J^P(E) = 0^- \qquad |M^0|^2 \sim a[\vec{p}_1 \cdot \vec{q}_2 \ BW(K_1^*) + \vec{p}_2 \cdot \vec{q}_1 \ BW(K_2^*)]^2 + (1 - a)\ [BW(K\bar{K})]^2$$

$$J^P(E) = 1^+ \qquad |M^1|^2 \sim a[\vec{p}_1 \ BW(K_1^*) + \vec{p}_2 \ BW(K_2^*)]^2 + (1 - a)\ [\vec{q}_\pi \ BW(K\bar{K})]^2$$

where $\vec{p}_1$, $\vec{p}_2$ are the momenta of the K in the centre of mass of the K_1^* and K_2^*;

$\vec{q}_1$, $\vec{q}_2$ are the momenta of the K's not associated with the K^*'s, in the E centre of mass;

$\vec{q}_\pi$ is the momentum of the pion in the E centre of mass;

$BW(K_1^*)$, $BW(K_2^*)$, $BW(K\bar{K})$ are Breit-Wigner amplitudes for K*(891) and a $K\bar{K}$ resonance (M = 990 MeV, Γ = 70 MeV).

The introduction of a $K\bar{K}$ resonance is not essential for the analysis of the E. The same results may be obtained with the introduction of a $K\bar{K}$ scattering length of 3 f.

The proportion of K* in the E decay is left as a free parameter in the fit. The experimental data for reactions (96) and (97) are treated independently.

Table 7

Reaction	$J^P(E)$	% K*	% ($K\bar{K}$)	% Back-ground	$\chi^2/\langle\chi^2\rangle$	Prob. %
$\bar{p}p \rightarrow E^0\pi^+\pi^-$	0^-	46 ± 3	42 ± 3	12 ± 6	52/34	2.0
	1^+	43 ± 3	33 ± 3	24 ± 6	63/34	0.2
	2^-	57 ± 3	25 ± 3	18 ± 6	76/34	0.0
$\bar{p}p \rightarrow E^0\pi^0\pi^0$	0^-	48 ± 4	52 ± 4	-	34/30	30.0
	1^+	64 ± 4	36 ± 4	-	44/30	5.0
	2^-	68 ± 4	32 ± 4	-	46/30	3.0

For the reaction $\bar{p}p \rightarrow E^0\pi^+\pi^-$, the presence of the background is due to the fact that we did not take into account interference effects between the two possible E's. It may also explain the poor fits obtained in this case.

4.7 Spin parity of the E - E production

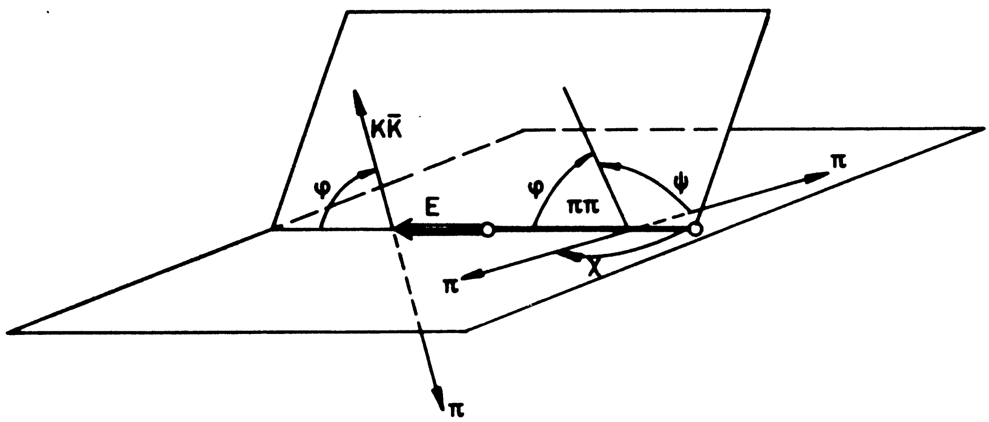
The E meson being produced in three-body reactions (96) and (97), the study of the production Dalitz plot and of specific angular distributions tells us something on the spin parity of the E, since the initial state ($\bar{p}p$) has well-defined quantum numbers.

Let us first remark that the comparison of the production rates of the E meson for reactions (96) and (97) leads to the conclusion that the $^1S_0$ initial state is preponderant in the initial state:

$$\bar{p}p \rightarrow E^0\pi^0\pi^0 \text{ is pure } C = +1, \text{ therefore pure } {}^1S_0 .$$

From the number of events observed in this channel, we can deduce the number of events due to $^1S_0$ in the $\bar{p}p \rightarrow E^0\pi^+\pi^-$ channel: it is found to represent at least 65% of the whole channel. However, in the following analysis we shall not use this information, and will leave the proportions of $^1S_0$, $^3S_1$ as a free parameter in the fits.

In addition to the mass spectra, the fit is performed on three angles defined in the following drawing:



When the production occurs in $^1S_0$, the orbital angular momentum of the dipion produced with the E must be even (C parity): we have limited the calculations to $\ell(\pi\pi) = 0$ and 2 (amplitudes α and β, respectively, in the following equations, with a phase ω).

When the production occurs in $^3S_1$, the orbital angular momentum of the dipion must be odd. We limit ourselves to $\ell(\pi\pi) = 1$.

For $J^P(E) = 0^-$, $^1S_0$:

$$W(\cos\varphi) = 1$$
$$W(\cos\psi) = 1$$
$$W(\cos\chi) = \alpha^2 + \frac{5\beta^2}{4}(3\cos^2\chi - 1)^2 + \sqrt{5}\,\alpha\beta\cos\omega \times (3\cos^2\chi - 1)\ .$$

For $J^P(E) = 0^-$, $^3S_1$:

$$W(\cos\varphi) = 1$$
$$W(\cos\psi) = 1$$
$$W(\cos\chi) = \sin^2\chi\ .$$

For $J^P(E) = 1^+$, $^1S_0$:

$$W(\cos\varphi) = 3\,\alpha^2\cos^2\varphi + \frac{3\beta^2}{10}(3 + \cos^2\varphi)$$
$$W(\cos\psi) = \alpha^2 + \frac{\beta^2}{2}(3\cos^2\psi + 1) + \sqrt{2}\,\alpha\beta\cos\omega \times (3\cos^2\psi - 1)$$
$$W(\cos\chi) = \alpha^2 + \frac{\beta^2}{2}(3\cos^2\chi + 1) + \sqrt{2}\,\alpha\beta\cos\omega \times (3\cos^2\chi - 1)\ .$$

For $J^P(E) = 1^+$, $^3S_1$:

$$W(\cos\varphi) = 1$$
$$W(\cos\psi) = \sin^2\psi$$
$$W(\cos\chi) = 1\ .$$

The results of the fit are the following:

Table 8

$J^P(E)$	$\chi^2/\langle\chi^2\rangle$	+ % $^1S_0$	% $\ell\pi\pi$ in $^1S_0$
0^-	44/37	83 ± 6	99^{+1}_{-6}
1^+	40/37	96 ± 6	10 ± 5
2^-	52/37	5 ± 5	20 ± 5

$J^P = 2^-$ does not give a good fit and assumes a too small percentage of $^1S_0$.

$J^P = 1^+$ gives a fit as good as 0^- but assumes a percentage of d-wave for the dipion produced with the E which does not seem to be acceptable. (The energy available in the dipion is limited to 500 MeV.) Indeed, 1^+ gives a poor fit when the d-wave is disregarded for the dipion.

The quantum numbers of the E are therefore likely to be $J^P = 0^-$.

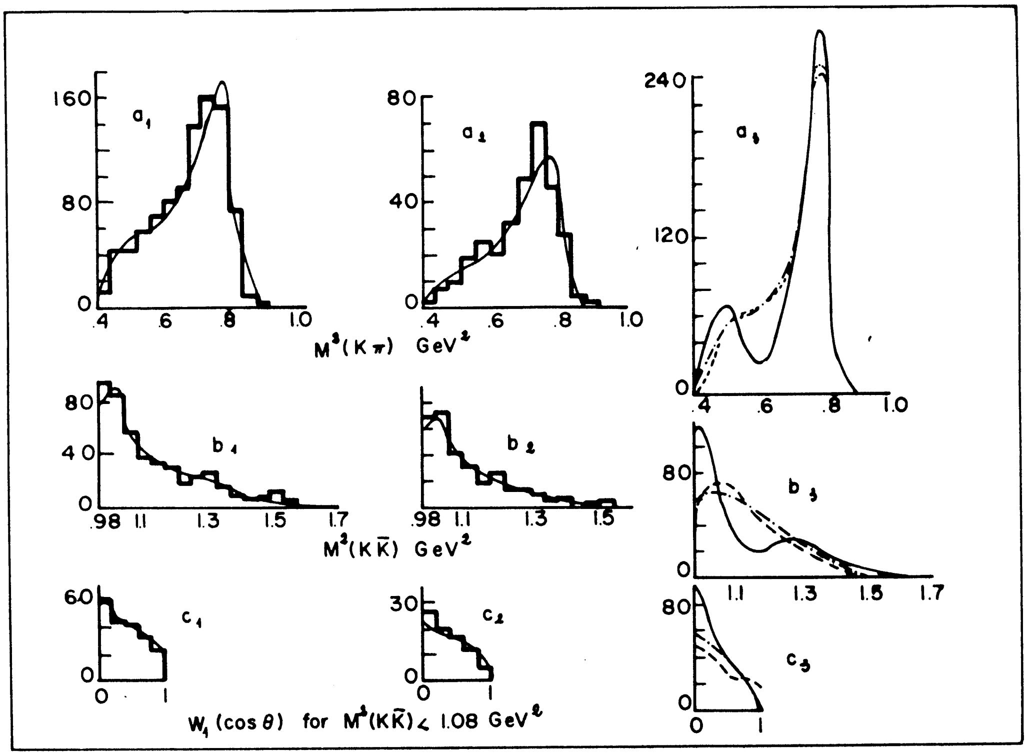


Fig. 10 Decay of the E meson[28].

a) : $(K\pi)$ effective mass spectra (two combinations per event).

b) : $(K\bar{K})$ effective mass spectra.

c) : W_1 (cos θ): angular distribution of the K mesons in the $(K\bar{K})$ centre of mass. For these angular distributions, an additional condition is required, namely: $M^2(K_1^0 K^\pm) < 1.08$ GeV^2.

a_1, b_1, c_1 refer to reaction (1): $\bar{p}p \to K_1^0 K^\pm \pi^\mp \pi^+ \pi^-$ when the $(K\bar{K}\pi)^0$ effective mass squared satisfy the conditions: 1.84 GeV^2 < $M^2(K_1^0 K^\pm \pi^\mp)$ < 2.14 GeV^2. This selection is made in order to obtain distributions corresponding as close as possible to the E decay.

a_2, b_2, c_2 refer to reaction (2): $\bar{p}p \to K_1^0 K^\pm \pi^\mp \pi^0 \pi^0$. The same conditions as for reaction (1) are requested.

a_3, b_3, c_3 refer to theoretical curves for $M^2(K\pi)$, $M^2(K\bar{K})$, W_1(cos θ) distributions corresponding to the decay $E \to K^*\bar{K}$ (and $\bar{K}^*K$) for $J^P(E) = 0^-, 1^+, 2^-$ (solid curves: 0^-; dotted curves: 1^+; mixed curves: 2^-). The curves drawn on a_1, b_1, c_1, a_2, b_2 and c_2 distributions correspond to the hypothesis $J^P(E) = 0^-$, with $E \to K^*\bar{K}$ (and $\bar{K}^*K$) : 50 %
$E \to (K\bar{K})\pi$: 50 %
where $(K\bar{K})$ stands for a resonance with M = 1000 MeV and Γ = 70 MeV.

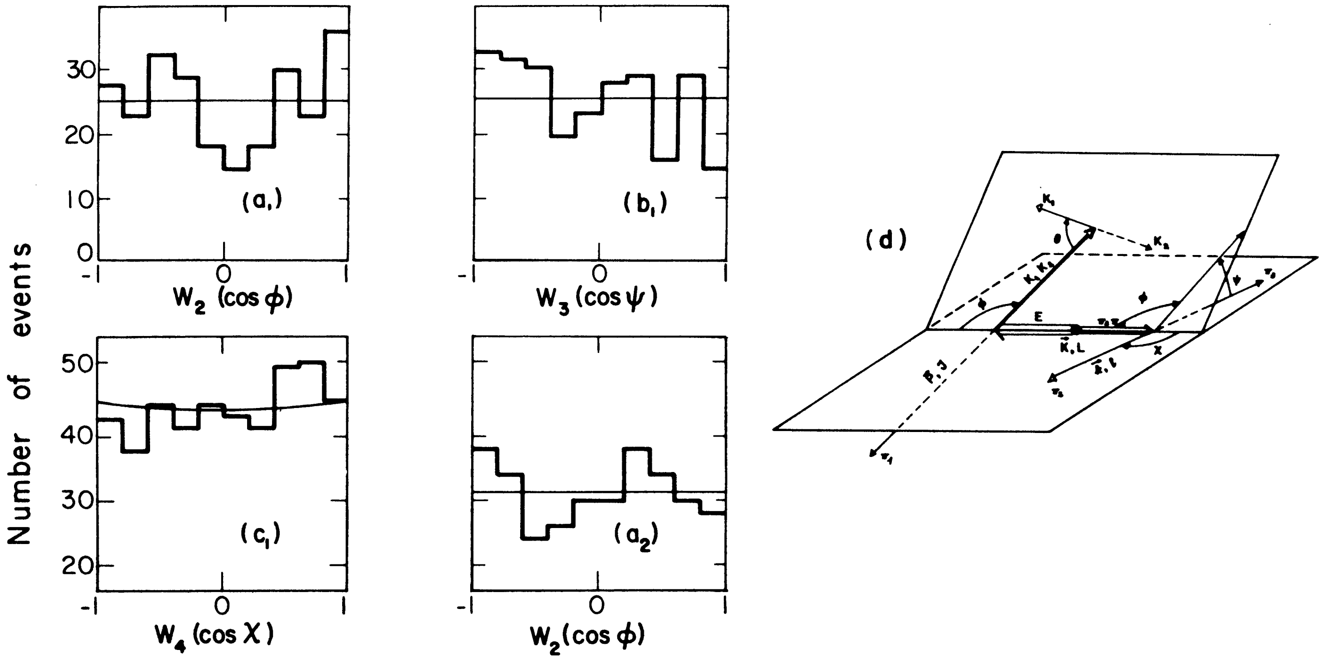


Fig. 11 E^0 production[28].

Angular distributions for the two step process $\bar{p}p \to E^0\pi\pi$, $E^0 \to (K\bar{K})\pi$, where E^0 is defined by $1.84\ GeV^2 < M^2(K\bar{K}\pi) < 2.14\ GeV^2$, and $(K\bar{K})$ by $M^2(K\bar{K}) < 1.08\ GeV^2$.

11a₁, 11a₂ : W_2 (cos φ) angular distributions for reaction (1) $\bar{p}p \to E^0\pi^0\pi^-$, and reaction (2) $\bar{p}p \to E^0\pi^0\pi^0$, respectively. φ is the decay angle of the E^0 into $(K\bar{K})$ and π, in the centre of mass of the E^0 (see Fig.11d).

11b₁ : W_3 (cos ψ) angular distribution for reaction (1) $\bar{p}p \to E^0\pi^+\pi^-$. ψ is the angle between the vector $\vec{p}$ characterizing the decay of the E into $(K\bar{K})$ and π, and the vector $\vec{k}$ characterizing the decay of the $(\pi_2\pi_3)$ dipion exterior to the E. ψ is measured in the total centre of mass.

11c₁ : W_4 (cos χ) angular distribution for reaction (1) $\bar{p}p \to E^0\pi^+\pi^-$. χ is the decay angle of the $(\pi_2\pi_3)$ dipion into π_2 and π_3, in the centre of mass of $(\pi_2\pi_3)$.

11d : summarizes the definitions used for angles, momenta, and angular momenta in the study of the E production.

The curves represented on Fig. 11a₁, b₁, c₁, a₂, correspond to the best fit, obtained for $J^P(E) = 0^-$.

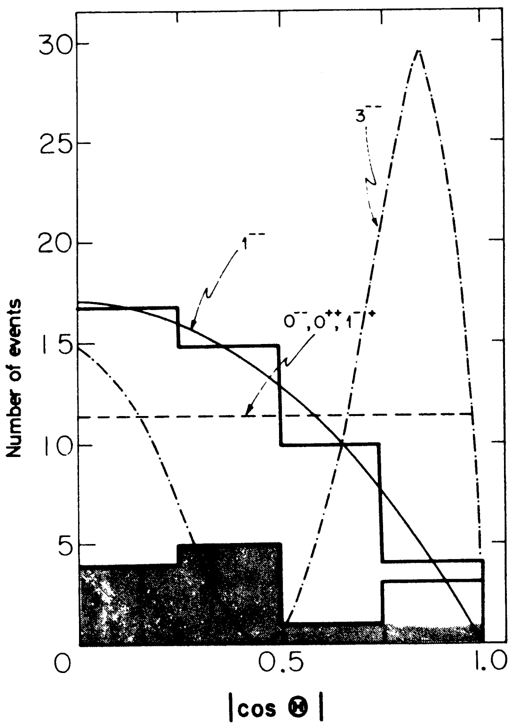


Fig. 12 Spin parity of the φ meson[29].

The φ mesons are produced in the reaction $\bar{p}n \to \pi^- \varphi$, with $\varphi \to K\bar{K}$. The annihilations occur at rest, therefore the quantum numbers of the initial state are relatively well known: $J^{PG} = 0^-$ or 1^{-+}

The histogram gives the angular distribution of the π^- with respect to the $\bar{K}$ in the φ c.m. system.

The curves correspond to various J^{PG} assignments for the φ for S-state $\bar{p}n$ annihilations.

The shaded histogram corresponds to the decay $\varphi \to K_1^0 K_2^0$, the white one to the decay $\varphi \to K^+K^-$.

The spin 1^{--} is strongly favoured.

5. THE DIPION SYSTEM

5.1 σ(400), ε(700)

The dipion system has been investigated intensively in the last four years.

Two well-established resonances have been reported: the ρ meson (M = 770 MeV, Γ = 140 MeV, $I^G J^P = 1^+1^-$) and the f^0 meson (M = 1254 MeV, Γ = 117 MeV, $I^G J^P = 0^+2^+$).

For the time being, there are no convincing experimental results for a scalar meson, although several "enhancements" have been observed, around 400 MeV (the "σ meson") and around 700 MeV (the "ε meson").

The direct observations of the dipion system very often suffer from the presence of other resonances [N*(1238), ρ(770)] which introduce the disturbing reflection and interference effects.

Samios et al.[30] have found a peak in the neutral dipion system at 395 MeV for events of the type:

$$\pi^- + p \to N + 4\pi$$

(N is for neutron or proton).

They also find that these reactions are strongly dominated by the ρ and N* productions.

Del Fabbro et al.[31] have also found a peak in the neutral dipion system at 379 MeV (Γ ~ 140 MeV) in a photoproduction experiment:

$$\gamma + p \to p + \pi^+ + \pi^- ,$$

but they cannot exclude the possibility of attributing this peak to the reflection of an isobar which may be formed by the excitation of the nucleon for a given photon energy. Kopelman et al.[32] also observed an enhancement in the neutral dipion system around 400 MeV for the reaction

$$K^- + p \to K^- \, p \, \pi^+\pi^- \quad ,$$

but here again there are so many other possible resonances [K*(891), N*(1238), N*(1515), K*(1310)?] that it is difficult to give a clear interpretation of the data.

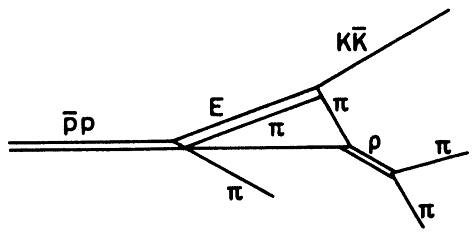
A perhaps more striking observation comes from the study of the decay of several resonances where an s-wave dipion is present.

The energy spectrum of the π^+ meson in the $\tau' \to \pi^+\pi^0\pi^0$ decay has been studied by Kalmus et al.[33]: they found a good interpretation if they introduced a Breit-Wigner factor for the $(\pi^0\pi^0)$ effective mass spectrum, as proposed by Brown and Singer[15]. However, for the mass and the width of this Breit-Wigner they found results incompatible with a similar analysis done for the $\eta \to \pi^+\pi^-\pi^0$ decay [Crawford[34]]. Moreover, we have already mentioned that the experimental branching ratios observed for the η meson ($\eta \to 3\pi^0$, $\eta \to \pi^+\pi^-\pi^0$) do not agree with the Brown and Singer argument.

$X^0 \to \eta\pi^+\pi^-$ decay also presents an anomaly for the dipion spectrum which could be explained by the presence of the σ^0 meson around 400 MeV, but the statistics are still too limited.

Recently, Baillon et al.[28] have observed that the neutral dipions ($\pi^+\pi^-$ and $\pi^0\pi^0$) produced in association with the E meson in $\bar{p}p$ annihilations do not follow phase space; the experimental spectrum may be understood if one introduces a Breit-Wigner factor characterized by $M \sim 450$ MeV and $\Gamma \sim 70$ MeV.

However, here again the observed deviation from phase space may be due to dynamical interference effects with the pion resulting from the E decay:



Indeed, the introduction of a "background" amplitude $\bar{p}p \to (K\bar{K})\rho\pi$ in coherence with the E production amplitude with the subsequent decay mode $E \to (K\bar{K})\pi$ leads to a good understanding of the dipion spectrum, with no need for the σ meson [Rogers[35]].

Finally, we should mention the results of several experiments done with low-energy π^- (below N* and ρ-meson thresholds):

$$\pi^- + p \to \pi^+ + \pi^- + n \quad ,$$

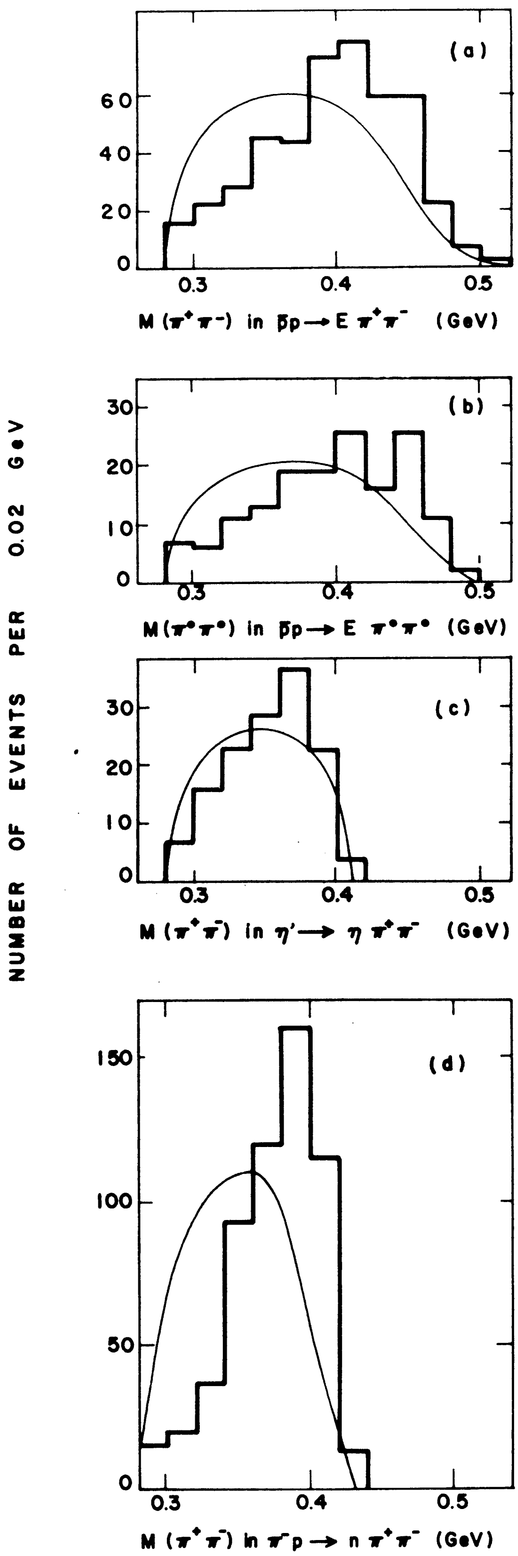


Fig. 13 The σ^0 meson.

$(\pi\pi)$ effective mass spectra for reactions in which the s-wave dipion is strongly favoured.

All these spectra show a shift towards the high mass limit and may be interpreted with the introduction of a resonance $(\pi\pi)$, $J^P = 0^+$, with $M \sim 450$ MeV.

a) $\bar{p}p \to E^0\pi^+\pi^-$

b) $\bar{p}p \to E^0\pi^0\pi^0$

c) $\eta' \to \eta\pi^+\pi^-$

d) $\pi^-p \to n\pi^+\pi^-$ at 360 MeV

where a deviation is often observed for the dipion at the upper limit of phase space, with little possibility, however, of deciding if this is due to a genuine s-wave $\pi\pi$ resonance or to another mechanism.

It seems reasonable to conclude that none of these results, taken separately, may be considered as unambiguous evidence for the existence of a σ^0 meson, although one cannot exclude the existence of a broad s-wave resonance in the 350-450 MeV region.

It has been suggested that a scalar resonance could be hidden by the presence of the ρ meson if its mass was in the range 650-850 MeV (the ϵ^0 meson). Indeed, one is still far from understanding the properties of the ρ^0 meson production and decay. Since this ρ meson is found to be aligned in most of the studied reactions, it is possible, in principle, to study the contribution of other dipion states by selecting the events according to their decay angle: for instance, for the reactions of the type

$$\pi + p \rightarrow p + \rho \quad ,$$

where the one-pion exchange mechanism dominates the ρ production (at least for small momentum transfers), the decay angular distribution of the ρ is $W(\vartheta) \sim \cos^2 \vartheta$.

For the $\bar{p}p$ annihilations at rest:

$$\bar{p}p \rightarrow \rho + \pi \ ,$$

the decay angular distribution of the ρ is $W(\vartheta) \sim \sin^2 \vartheta$.

There have been some experimental indications that the dipion effective mass squared is shifted from the central mass value of the ρ when such a selection on the decay angle is operated [Selove[36]]. However, all experimental results do not agree on this fact [Jacobs[37]]. It is not even clear that, if there is a mass shift, it occurs only for the neutral dipion system.

The experimental situation for the $\pi^0\pi^0$ system (for which there is no T = 1 contribution) is not clear [Feldman et al.[38], Cohn et al.[39]] but gives rather negative results with regard to the existence of a possible ϵ^0 meson.

In these conditions, it is hopeless to attempt a spin parity analysis of this elusive ϵ meson.

The natural assumption is to assume T = 0, but there are a few experimental results which seem to indicate that the T = 2 dipion amplitude may be large near the threshold, and could still be sufficiently important around 700 MeV to allow detectable interference effects with the ρ meson.

Let us conclude that the complete understanding of the dipion behaviour in the 300-800 MeV range, apart from the T = 1 $J^P = 1^-$ amplitude, is still missing. No clear conclusions can be drawn from the present experimental situation concerning the existence of new resonances (σ and ϵ mesons), and it will be difficult to improve this situation if the width of these hypothetical resonances is smaller than the experimental resolution (Γ < 15 MeV) or larger than 100 MeV.

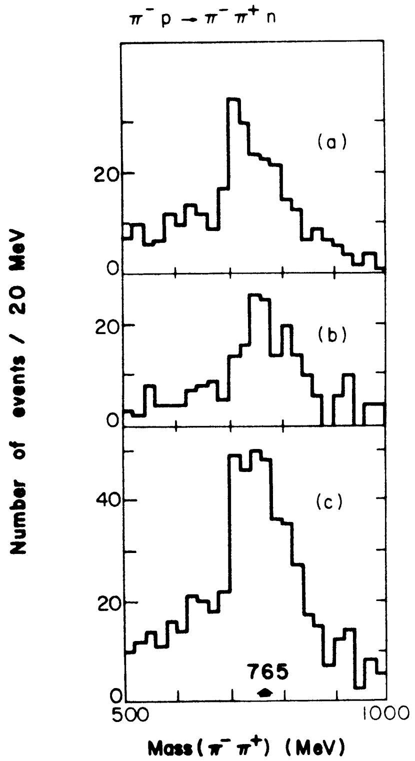


Fig. 14 The ϵ^0 meson.

The ρ meson being produced by the OPE mechanism, its decay angular distribution follows the $\cos^2 \vartheta$ rule. By selecting events for which $\cos \vartheta$ is small, one favours the study of the "non-ρ events".

Thus, Selove obscures a small shift of the mass of the resonance, and tentatively concludes that there is present another dipion resonance, the ϵ^0 meson (a).

However, Jacobs, with comparable statistics, does not observe this shift (b).

The sum of these two results (c) does not show a significant effect. (The central value of the mass of the ρ meson is 765 MeV.)

5.2 <u>The f^0 meson</u>

A peak in the neutral dipion system has been observed by Selove et al.[40)] and Veillet et al.[41)] at M = 1254 MeV with Γ = 117 MeV.

The reactions: $\pi^- + p \to n + \pi^+ + \pi^-$

and $\pi^- + p \to p + \pi^- + \pi^0$

were analysed, and the comparison of the peak observed in the neutral $(\pi^+\pi^-)$ with the absence of peak in the $(\pi^-\pi^0)$ system was taken as evidence for I = 0 for the f^0 meson.

However, this conclusion may be wrong if the production mechanism for the I = 1, I_z = 0 dipion system is different from the production mechanism for the I = 1, I_z = -1 dipion system. It was therefore essential to find the $(\pi^0\pi^0)$ decay mode of the f^0 meson, which is forbidden for I = 1.

This $\pi^0\pi^0$ decay mode has been indirectly "observed" in a missing-mass spectrum:

$$\pi^+ + d \to p + p + \text{neutrals},$$

and the comparison of the peak observed with the one observed in the $(\pi^+\pi^-)$ effective mass spectrum for the reaction

$$\pi^+ + d \to p + p + \pi^+ + \pi^-$$

gave the expected ratio

$$\frac{f^0 \to \pi^0\pi^0}{f^0 \to \pi^+\pi^-} = \frac{1}{2} \quad \text{for } I = 0 .$$

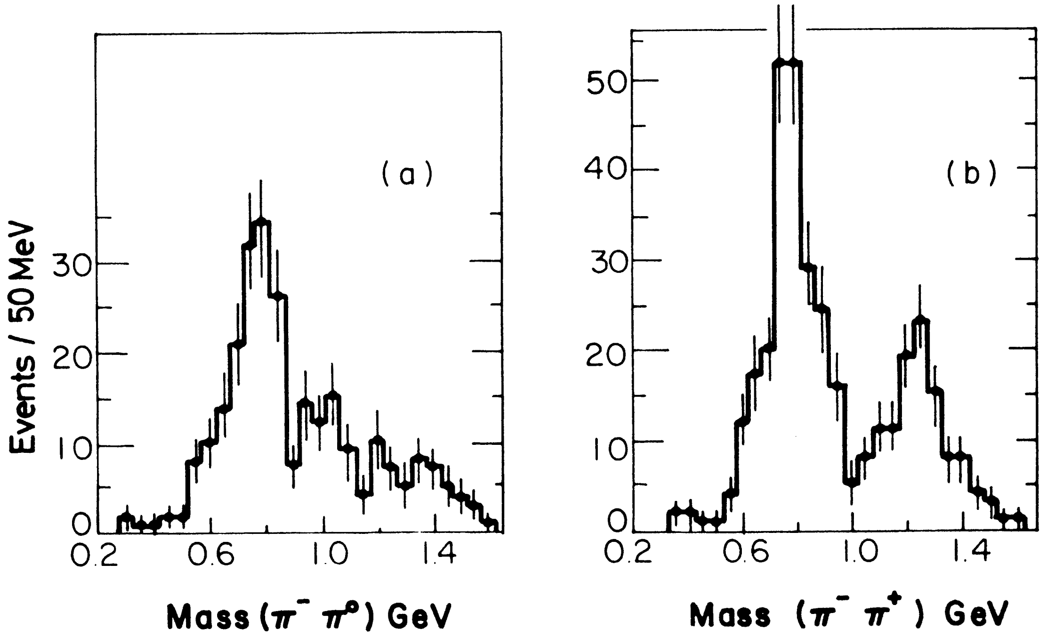


Fig. 15 ($\pi\pi$) effective mass spectrum.

a) for the reaction $\pi^-p \to \pi^-\pi^0 p$, which shows a large accumulation at 765 MeV, the ρ meson.

b) for the reaction $\pi^-p \to \pi^+\pi^- n$, which shows two enhancements, at 765 MeV and at 1254 MeV, corresponding to the ρ^0 and the f^0 meson, respectively. [(Selove et al.[40])].

We now have direct observations of the $\pi^0\pi^0$ decay mode with spark chamber experiments [Wahlig et al.[42]].

The spin parity analysis of the f^0 meson is not completely satisfactory. Since I = 0, it must be of the series $J^P = 0^+, 2^+, 4^+ \dots$.

The f^0 produced with small momentum transfer is found to be "aligned". An Adair analysis is therefore possible. One finds that it is necessary to include terms in $\cos^4 \vartheta$ to explain the decay angular distribution; this is, of course, a strong evidence against $J^P = 0^+$. However, the complete decay angular distribution does not follow the $(3 \cos^2 \vartheta - 1)^2$ rule expected for a spin 2^+ particle. Nearly all experimental results agree on the necessity of adding a relatively large s-wave background to the spin 2^+ resonance, unless one introduces a large d-wave background; in this last case, it is even impossible to exclude 0^+ assignment for the f^0 meson.

The new results of Wahlig et al.[42] clearly exclude $J^P = 4^+$ assignment but also present this ambiguity between 0^+ and 2^+, although 2^+ gives a slightly better fit.

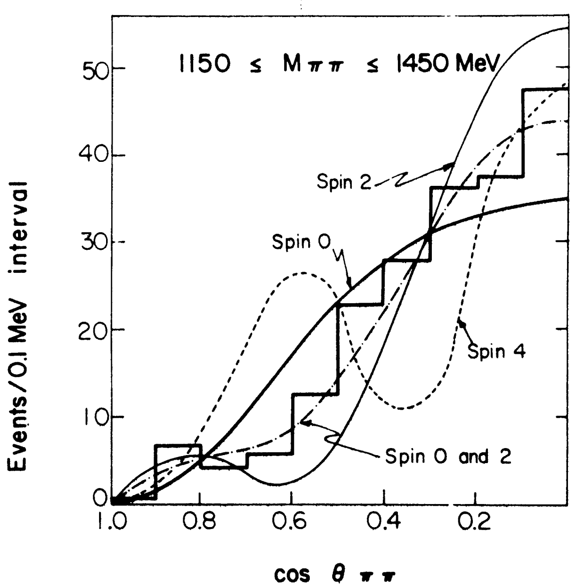


Fig. 16 $f^0 \to \pi^0\pi^0$: Data obtained by Wahlig et al.[42] with the reaction $\pi^-p \to n\pi^0\pi^0$ at 10 GeV/c.

The decay angular distribution shows the difficulty to assign a well-defined spin to the f^0 (the best fit is obtained for a combination of the 0^+ and 2^+ theoretical distributions). The non-isotropy given for 0^+ results from the experimental conditions.

With these quantum numbers, the f^0 meson can decay into a $K\bar{K}$ pair (K^+K^- and $K_1^0K_1^0$). As we shall see in Section 7, the $K_1^0K_1^0$ effective mass spectrum observed for several reactions and with different techniques often shows a large enhancement in the mass region of the f^0. Unfortunately, it is difficult to attribute unambiguously this enhancement to the $K_1^0K_1^0$ decay mode of the f^0 since several resonances are probably present in the same mass region.

No K^+K^- decay mode has been observed.

Thus, for the $K\bar{K}$ decay mode of the f^0, one can only give an upper limit:

$$\frac{\Gamma(K\bar{K})}{\Gamma(\pi\pi)} < 3\% \quad .$$

5.3 The g meson

It is now well established that the neutral dipion spectrum shows an enhancement around $M \sim 1650$ MeV [G. Goldhaber[43]]

However, the nature of this enhancement (called the g meson) is not well understood. In particular, the forward-backward asymmetry for the $\pi^+\pi^-$ system does not show any tendency to fall to zero at 1650 MeV as it does for the ρ^0 and f^0 masses: indeed, this asymmetry rises continuously with the incident energy, as it is expected for these strongly peripheral interactions.

An enhancement is also observed for the charged dipion in the same mass region. This observation comes from the study of the reactions:

$$\pi^\pm + p \to \pi^\pm + \pi^0 + p \text{ at 6 and 8 GeV/c [Crennell et al.}^{44)}] \; .$$

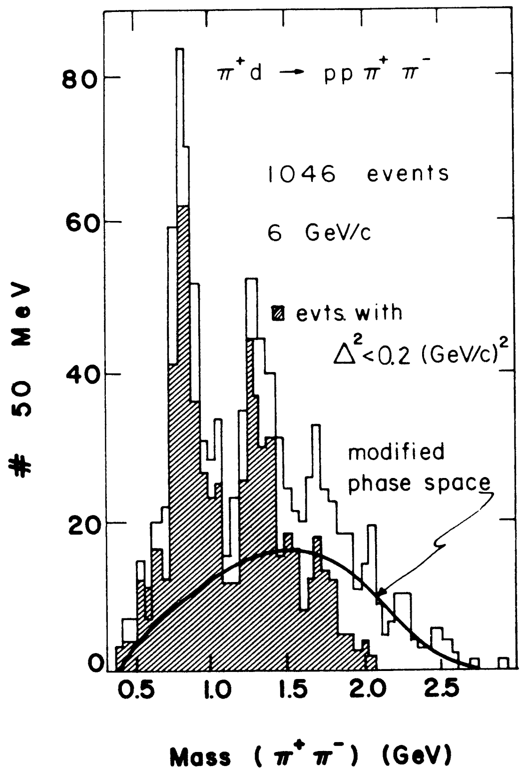


Fig. 17 The g meson[45].

$(\pi^+\pi^-)$ effective mass spectrum for the reaction

$$\pi^+ d \to pp\pi^+\pi^-$$

at 6 GeV/c. The shaded area corresponds to a quadrimomentum transfer to the $(\pi^+\pi^-)$ system lower than 0.2 (GeV/c)^2.

Three enhancements may be observed: the ρ^0(765 MeV), the f^0(1250 MeV), and the g^0(1675 MeV).

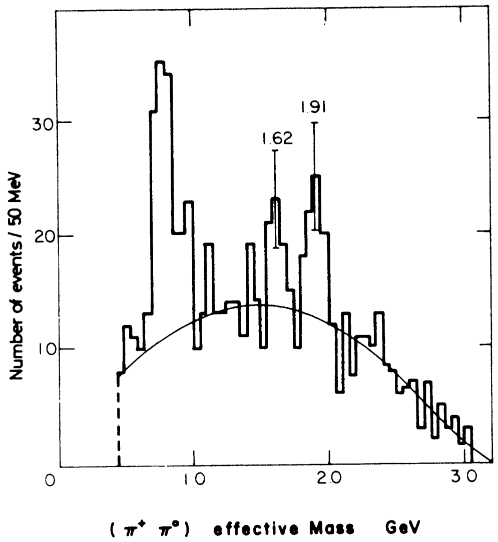


Fig. 18 $(\pi^+\pi^0)$ effective mass distribution

$$\pi^+ p \to p\pi^+\pi^0$$

at 8 GeV/c[46].

The curve is drawn by hand to fit the regions outside possible $(\pi^+\pi^0)$ enhancements.

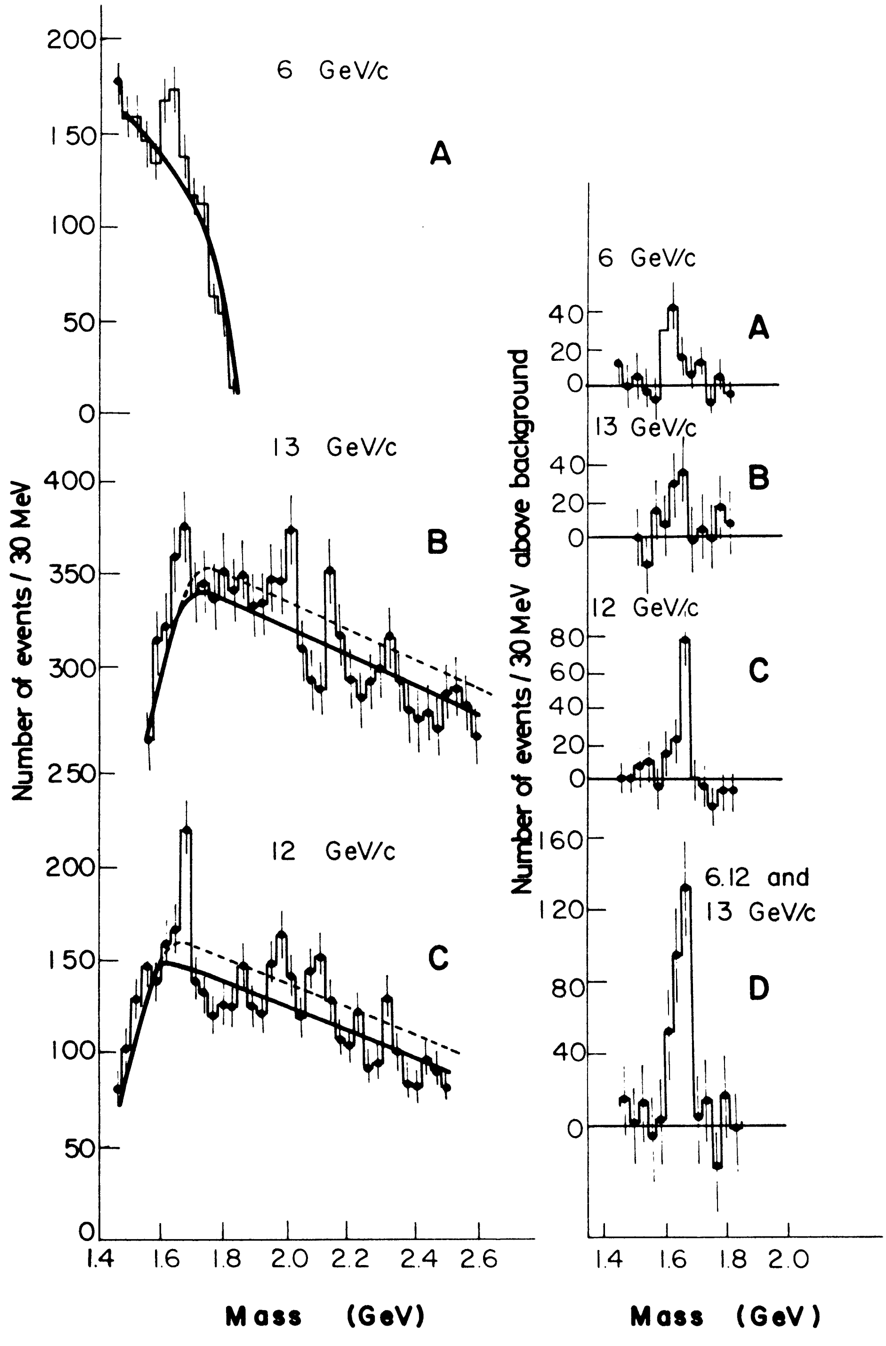


Fig. 19 Missing mass spectra for the recoil proton in $\pi^- p \to pX^-$, taken at three different pion momenta: 6, 12 and 13 GeV/c. The dotted lines are normalized to all events. The heavy lines are normalized to all events minus peaks higher than 3 s.d.

The histograms on the right show the same events, background subtracted.

D gives the sum of the 3 contributions[47].

The missing-mass spectrometer experiment [Focacci et al.[48]] also indicates the presence of enhancements in this mass region, without the possibility, however, of deciding if it is a G = +1 or G = -1 object.

To help to understand the nature of this enhancement, it is desirable to study other possible "decay modes", like the $K\bar{K}$ system. For the time being, adding together all the available information on the $\pi\pi$ system, one may be tempted to propose the existence of a new object with $I^G = 1^+$, M = 1630 MeV, $\Gamma \sim 150$ MeV (sometimes called the g meson).

An analysis of the decay angular distribution[44] indicates a spin larger than one. Since I = 1, J = 2 is excluded and one concludes J = 3 or higher.

6. THE TRIPION SYSTEM

In addition to the well-known $I^GJ^P = 0^-1^-$ ω^0 meson, four mesons decaying into three pions have recently been proposed: the $H^0(975)$, the $A_1(1080)$, the $A_2(1300)$, and the $A_3(1640)$.

6.1 The H^0 (975) meson

The H^0 meson is observed in the neutral tripion system around 975 MeV for the reactions:

$$\pi^+ + p \to (3\pi)^0 + N^{*++} \text{ at } 4 \text{ GeV/c } ^{49)}$$

$$\pi^+ + d \to (3\pi)^0 + p + p \text{ at } 3.65 \text{ GeV/c } ^{50)},$$

but it seems to be produced for a relatively narrow energy range of the incident particle. For instance, it is not observed at 3.5 GeV/c, nor is it at 5.1 GeV/c.

The width seems to be large: $\Gamma \sim 90$ MeV. There are some indications that this H meson is associated to the ρ meson, suggesting a decay mode $H \to \rho\pi$. However, the H meson production may be in competition with the X^0 (see Chapter 4.3) and the $\varphi \to \pi^+\pi^-\pi^0$. Moreover, the H meson is produced above a large background. If, however, one takes seriously the $\rho\pi$ mode of the H meson, then I(H) = 0 is favoured. More precisely, the branching ratios $\rho^+\pi^-$: $\rho^0\pi^0$: $\rho^-\pi^+$ satisfy the 1 : 1 : 1 relation expected for I = 0 (whereas it should be 1 : 0 : 1 for I = 1 and 1 : 4 : 1 for I = 2). Here we have assumed that the decay mode in which a ρ^0 is observed is more likely to be $\rho^0\pi^0$ than $\rho^0\gamma$.

One should, however, consider the possibility that the "$\rho^0\pi^0$" events are indeed due to the "$\rho^0\gamma$" decay mode of the X^0, which is known to be produced in these reactions (the $\rho^0\pi^0$ and $\rho^0\gamma$ are indistinguishable from the experimental point of view).

Therefore, a direct determination of the isotopic spin of the H meson from its production properties is welcome. The comparison of the charged tripion with the neutral one again favours I = 0.

The present status of the experimental situation is such that spin parity analysis of the H^0 is difficult. However, such an analysis has been attempted by Benson et al.[50], using the Berman-Jacob technique described in Section 3. No clear choice is possible for J^P; however, both this angular analysis and the distribution of events on the Dalitz plot favour 1^+ assignment.

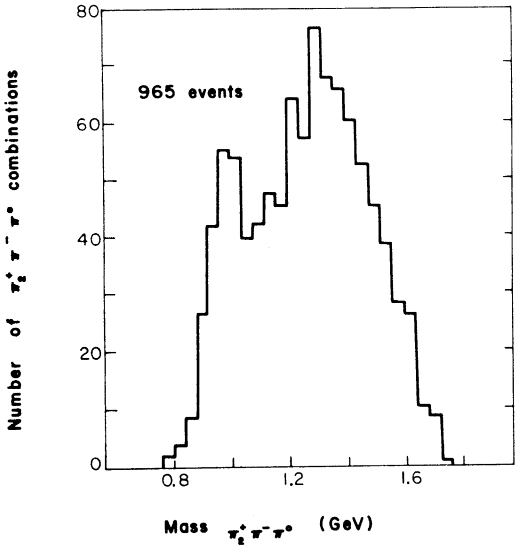


Fig. 20 The H meson.

$(\pi_2^+\pi^-\pi^0)$ effective mass spectrum for the reaction

$\pi^+p \to p\pi_1^+\pi_2^+\pi^-\pi^0$ at 4 GeV/c[49])

when $p\pi_1^+$ is in the N* region and when at least one of the combinations $\pi_2^+\pi^-$, $\pi_2^+\pi^0$, $\pi^-\pi^0$ is in the ρ region.

The presence of $\rho^0\pi^0$ combinations exclude I = 1 for the tripion system, therefore exclude the A_1.

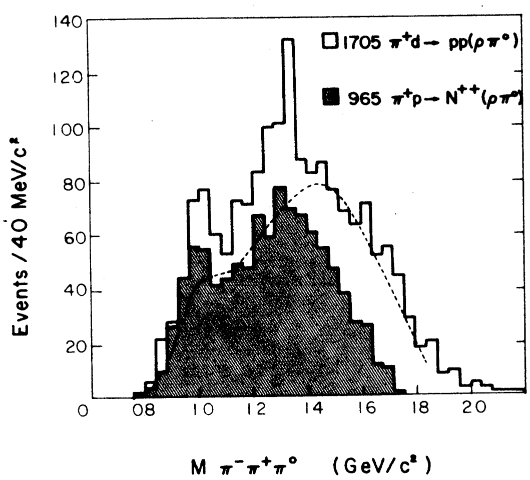


Fig. 21 $(\pi^+\pi^-\pi^0)$ effective mass spectrum for the reaction

$\pi^+d \to p(p)\pi^+\pi^-\pi^0$ at 3.65 GeV/c

when at least one of the combinations $\pi^+\pi^-$, $\pi^+\pi^0$, $\pi^-\pi^0$ is in the ρ region[50]).

The shaded histogram is the reproduction of the results shown in figure 20.

6.2 The $A_1(1080)$ meson

The A mesons A_1 and A_2 appeared first as a large accumulation of events for the $(\rho\pi)$ systems in the 1 GeV - 1.4 GeV mass range.

This large enhancement was soon interpreted as being due to the presence of two resonances: the A_1, with M = 1080 MeV, and the A_2, with M = 1300 MeV.

The A_1 and A_2 mesons have been observed for a wide range of energy of the incident particle:

$$\pi^{\pm} + p \rightarrow p + \pi^{\pm} + \pi^{+} + \pi^{-}$$

from 2.75 GeV/c to 16 GeV/c.

However, while the A_2 is nearly always observed, the A_1 is not always clearly seen: at present, the best evidence seems to come from the 8 GeV π^+p experiment[51], but for instance at 7 GeV it does not show up at all!

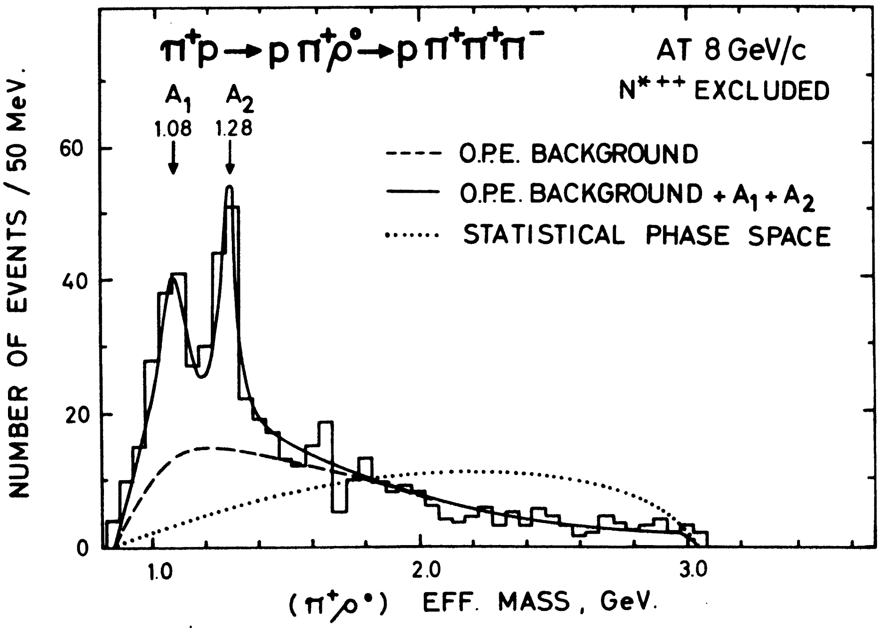
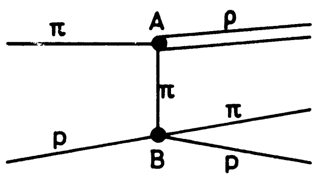


Fig. 22 $(\pi^+\rho^0)$ effective mass spectrum for the reaction: $\pi^+p \rightarrow p\pi^+\rho^0$ at 8 GeV/c. A cut has been applied to remove most of the events giving an $N^{*++}(1238)$. Two peaks are visible, at 1080 MeV and at 1280 MeV. They are identified to the A_1 and A_2 meson, respectively [51].

In addition to these rather surprising fluctuations for its production cross-section, the A_1 suffers from another serious difficulty: it is usually found above a very important background which may have misleading consequences; it has even been suggested that the A_1 is not a resonance but just an effect of the one-pion exchange mechanism, when this mechanism is treated to its extreme consequences. This is the so-called "Deck effect".

The Deck effect may be visualized by the diagram:



where at A, the ρ meson is produced with the characteristics of the OPE mechanism [small momentum transfer, decay angular distribution $W(\vartheta) \sim \cos^2 \vartheta$].

At B, the virtual exchange pion is supposed to scatter on the proton with the angular distribution characteristic of the real πp scattering at the same total energies (strong forward peak).

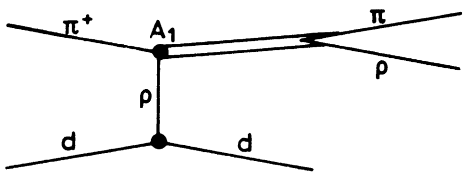
Moreover, the "decay" planes of the ρ meson and of the πp system must be independent since the exchanged particle is a pseudoscalar one (test of Treiman-Yang).

All these predictions are relatively well satisfied for the $(\pi\rho)$ system around 1 GeV.

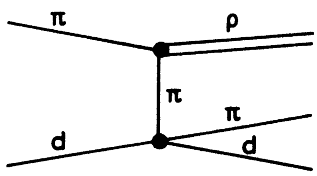
Moreover, there is direct experimental evidence for the presence of the Deck effect in similar reactions: for instance, in the reaction

$$\pi^- + d \rightarrow d + \pi^+ + \pi^- + \pi^- .$$

Where the deuterium is preserved, as such, in the final state, the A_1 production is probably strongly reduced, since a ρ-meson exchange is forbidden (by isotopic spin conservation):



whereas the Deck effect is allowed and favoured (small momentum transfer to the d)



There is experimental evidence for a broad accumulation of events around 1.1 GeV for such a reaction.

However, it is felt that the Deck mechanism cannot explain the relative narrowness of the A_1 enhancement. Deutschmann et al.[52] have computed the Deck effect for different incident energy; they find it can be "represented" by a Breit-Wigner always centred at 1.1 GeV, but with a width which depends on the incident energy :

$\Gamma \sim 350$ MeV for 3.65 GeV/c
$\Gamma \sim 500$ MeV for 8 GeV/c
$\Gamma \sim 700$ MeV for 16 GeV/c .

These widths are much larger than the width attributed to the A_1, when this one is measurable:

$$\Gamma(A_1) \sim 130 \pm 50 \text{ MeV} .$$

Another difficulty with the A_1 is its non-observation in the neutral mode $(\rho^{\pm}\pi^{\pm})$.

One possible way to decide if the A_1 is primarily due to the Deck mechanism or is a genuine $(\rho\pi)$ resonance is to study the reaction

$$\pi^+ + d \rightarrow p + p + \pi^+ + \pi^- + \pi^0 .$$

For the Deck mechanism, one does not expect to see an enhancement in the $\rho^-\pi^+$ system (it would correspond to the exchange of a double-charged particle). For an I = 1 resonance, it is the $(\rho^0\pi^0)$ system which should not show an enhancement. For the time being, the experimental results do not bring a clear answer.

In these conditions, the spin parity analysis of the A_1 meson is far from being well established.

The study of the production of the A_1, in a heavy-liquid bubble chamber and at a high energy, indicates that the production takes place via the exchange of pure, orbital, angular momentum between the incident pion and the target. If this is confirmed, it means that the spin parity of the A_1 meson is to be found in the series (coherent production):

$$0^-, 1^+, 2^- \dots .$$

Some angular distributions seem to exclude 0^-.

1^+ gives a good interpretation of the experimental results, but one still needs some admixture of negative parity to get a good fit [Allard et al.[53]].

Preliminary results on the hydrogen bubble chamber experiment[51] also favour 1^+ assignment, 2^- being the next best assignment.

The non-observation of $A_1 \rightarrow K\bar{K}$ may also be taken as a weak evidence for such assignments.

However, it seems of prime importance to establish unambiguously the existence of the A_1 meson, preferably in reactions less or not at all sensitive to the Deck type of effects; this may be the case for the $\bar{p}p$ annihilations into pions.

6.3 The A_2(1300) meson

The $(\rho\pi)$ systems show an enhancement around 1300 MeV. This has been observed in the singly charged and in the neutral system. It is therefore assumed that it is due to an I = 1 resonance, called the A_2 meson.

The A_2 meson, as the A_1, has been mainly studied in pion-nucleon interactions from 2.75 to 16 GeV/c.

Some of these experiments have also shown an enhancement around 1300 MeV in the $K\bar{K}$ system. It may be attributed to the $A_2 \rightarrow K\bar{K}$ decay mode.

However, the experimental results disagree, in general, on the central value of the mass of the A_2, on the $K\bar{K}/\rho\pi$ branching ratio, and on the spin parity assignments.

These disagreements suggest a more complicated structure such as the existence of two resonances of comparable masses but different quantum numbers.

Let us first consider the experimental situation for the $(\rho\pi)$ system.

The results on the central value of the mass cover the 1270–1335 MeV range, with repeated indications for a splitting of the enhancement into two peaks centred at ~ 1285 and ~ 1320 MeV.

The results on the width also extend over a large range, going from 50 MeV to 150 MeV. This spread of the results may also be due, in fact, to the presence of two resonances which are produced with cross-sections depending on the total energy.

Since the A_2 is produced with a large background (at least 50%), the spin parity determination depends very much on the assumptions made for the structure of this background.

A good fraction of this background is due to the peripheral production of the ρ meson, and the estimation of its reflection on the decay angular distributions makes the choice difficult between different spin parity assignments.

Three assignments are considered as being possible:

$$1^+,\ 2^-,\ 2^+ .$$

The non-observation of the A_2 meson in the heavy-liquid bubble chamber work at high energy (already mentioned in the discussion of the A_1 meson) may be taken as weak evidence that the spin of the A_2 is not of the series:

$$0^-,\ 1^+,\ 2^- \ldots ,$$

if the tripion is produced in these reactions by exchange of orbital angular momentum between the incident pion and the target nucleus.

Moreover, an enhancement is observed in many reactions ($\pi^\pm p$, $\bar{p}p$ annihilations) in the charged $K_1^0K^\pm$ effective mass spectrum around 1300 MeV.

If this $K_1^0K^\pm$ resonance is attributed to the A_2 meson, it leads to a unique solution for the spin parity determination. Since $G(A_2) = -1$ ($A_2 \to 3\pi$), $L(K\bar{K})$ must be even [$G(K\bar{K}) = (-1)^{I+L} = (-1)^{1+L}$] . $L = 0$ is excluded since a scalar particle cannot decay into three pseudoscalars. Therefore $L = 2$ (or $4 \ldots$) and

$$J^P(A_2) = 2^+ .$$

This conclusion is indeed confirmed by the study of the decay angular distribution of the $K\bar{K}$ system.

Moreover, some experiments have shown the existence of a $K_1^0K_1^0$ enhancement at the same mass value. This is again in favour of 2^+ assignment.

However, it is always dangerous to try to determine the spin parity of a particle, using different decay modes simultaneously.

In particular, for the A_2 meson the fluctuations observed for the central value of its mass, when going from one reaction to another one, confuse the identification of the $(K\bar{K})$ system with that of the $(\rho\pi)$ one.

For the resonance observed in the $K_1^0K^\pm$ system, Chung et al.[54] give a mass $M = 1320$ MeV, whereas the $(K\bar{K})$ enhancement observed in $\bar{p}p \to K\bar{K}\pi$ at rest is centred at $M = 1280 \pm 10$ MeV. (These two experiments reach the same conclusion for the spin: $J^P = 2^+$.)

Another difficulty lies in the branching ratio of the A_2 meson:

$$R = \frac{\Gamma(K\bar{K})}{\Gamma(\rho\pi)} .$$

A few experiments have led to the determination of this ratio. It is of the order of 5% for the A_2 observed in π^-p reactions around 3 GeV/c. It would be interesting to study this ratio at higher energies and for other reactions.

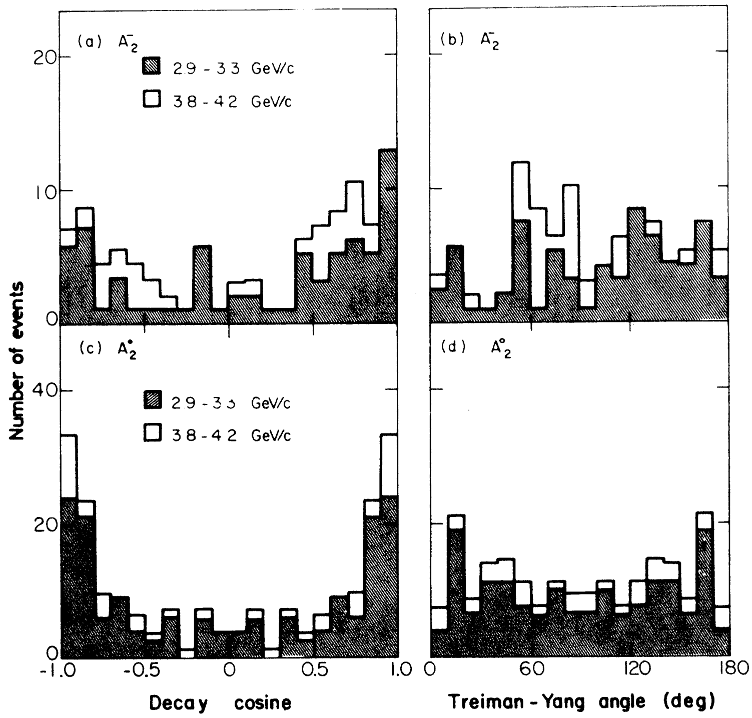


Fig. 23 Spin of A_2 meson[54] decay angular distribution of the A_2 events, in the A_2 c.m.s. and Treiman-Yang angle. The observed decays are $A_2 \to K^-K_1^0$ (a) and (b), and $A_2^0 \to K_1^0K_1^0$ (c) and (d).

It might turn out that this ratio depends on the reaction considered: this could perhaps be the key to the problem of the presence of two neighbouring resonances.

For the time being, we shall conclude that there is at least one 2^+ resonance with a mass of 1300 ± 20 MeV observed in the $K\bar{K}$ system. The $\rho\pi$ enhancement observed in the same mass region may be attributed (at least partly) to the same resonance. The present experimental situation is not clear, and one should not exclude the possibility of the presence of at least another resonance in this mass region.

6.4 The A_3(1640)meson

A compilation on the $(\pi^+\pi^-\pi^+)$ mass spectrum done by Ferbel[55] for the reaction:

$$\pi^+ + p \to \pi^+ + \pi^- + \pi^+ + p$$

at 7, 8 and 8.4 GeV/c shows an enhancement at 1640 MeV.

This enhancement is also seen by Guszavin et al.[56] for the reaction:

$$\pi^- + p \to \pi^- + \pi^+ + \pi^- + p \quad \text{at 4.7 GeV/c .}$$

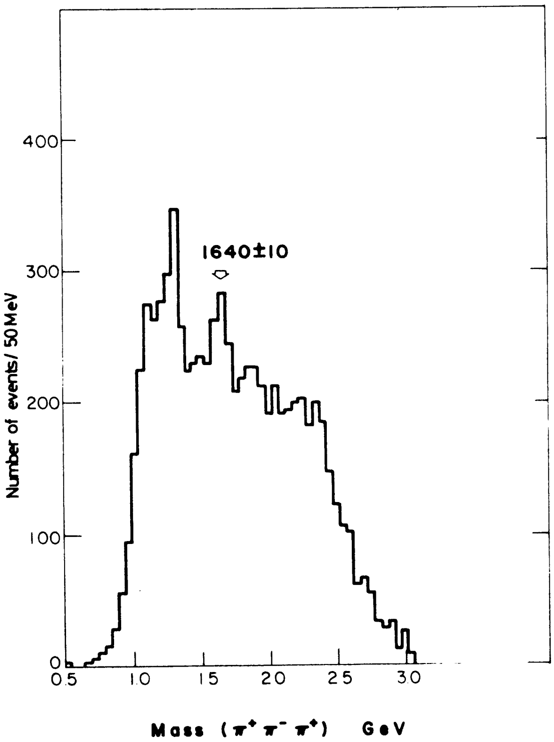


Fig. 24 The $A_3(1640)$[55].

Compilation of $(\pi^+\pi^-\pi^0)$ effective mass spectrum for different reactions.

Two enhancements are visible, at 1300 MeV (the A_2 meson), and at 1640 MeV.

The enhancement seems too narrow to be attributed to the reflection of an N* (Nπ, N$\pi\pi$ resonances). It does not seem to be associated with the ρ meson.

Nothing is known about the spin and parity of this object, but if it is a resonance, I = 1, G = -1.

7. THE $K\bar{K}$ SYSTEMS

We have already used the $K\bar{K}$ decay mode of the A_2 meson for the determination of its spin and parity. This is a rather powerful method, for the resonances of which the $K\bar{K}$ decay mode is the preponderant one since the quantum numbers of a $K\bar{K}$ system are restricted by the nature of the K^0's decays:

An I = 1 $K\bar{K}$ system leads to the following possibilities:

for $I_z = \pm 1$: $K^0K^{\pm}$, with $J^{PG} = 0^{+-}, 1^{-+}, 2^{+-} \ldots$

for $I_z = 0$: K^+K^-, $K_1^0K_1^0$, $K_2^0K_2^0$, $K_1^0K_2^0$, with K^+K^- : $J^{PG} = 0^{+-}, 1^{-+}, 2^{+-} \ldots$

$K_1^0K_1^0$ and $K_2^0K_2^0$: $J^{PG} = 0^{+-}, 2^{+-}, 4^{+-} \ldots$

$K_1^0K_2^0$: $J^{PG} = 1^{-+}, 3^{-+} \ldots$.

An I = 0 $K\bar{K}$ system leads to the following possibilities:

K^+K^- with $J^{PC} = 0^{++}, 1^{--}, 2^{++} \ldots$

$K_1^0K_1^0$ and $K_2^0K_2^0$ with $J^{PC} = 0^{++}, 2^{++} \ldots$

$K_1^0K_2^0$ with $J^{PC} = 1^{--}, 3^{--} \ldots$.

Apart from the well-known φ meson, there are at least three "objects", mainly observed in the $K\bar{K}$ system, which we would like to describe: the $K\bar{K}(990)$, the $K\bar{K}(1050)$, the $f'(1500)$.

7.1 The $(K\bar{K})$ structure near threshold

The structure of the $K\bar{K}$ system near threshold is rather complicated.

In the I = 0 channel, two resonances with opposite C-parity have been observed: the $\varphi(1020)$ and the $S^*(1050)$.

In the I = 1 channel, several experiments (most of them done on $\bar{p}p$ annihilations) have shown a strong accumulation in the $K_1^0K^\pm$ mass spectrum near threshold.

Let us describe the results obtained for the annihilation at rest into three bodies:

$$\bar{p}p \rightarrow K_1^0K^\pm\pi^\mp \ .$$

Apart from the $K^*(891)$'s and the $A_2(1280)$ observed in the $(K\pi)$ and $(K\bar{K})$ systems, respectively, the population of the Dalitz plot is far from being uniform: it shows a hole in the centre, a slight accumulation towards the edges corresponding to the threshold for the $(K\pi)$ systems, but over-all it shows a relatively sharp peaking of the density near the $(K\bar{K})$ threshold.

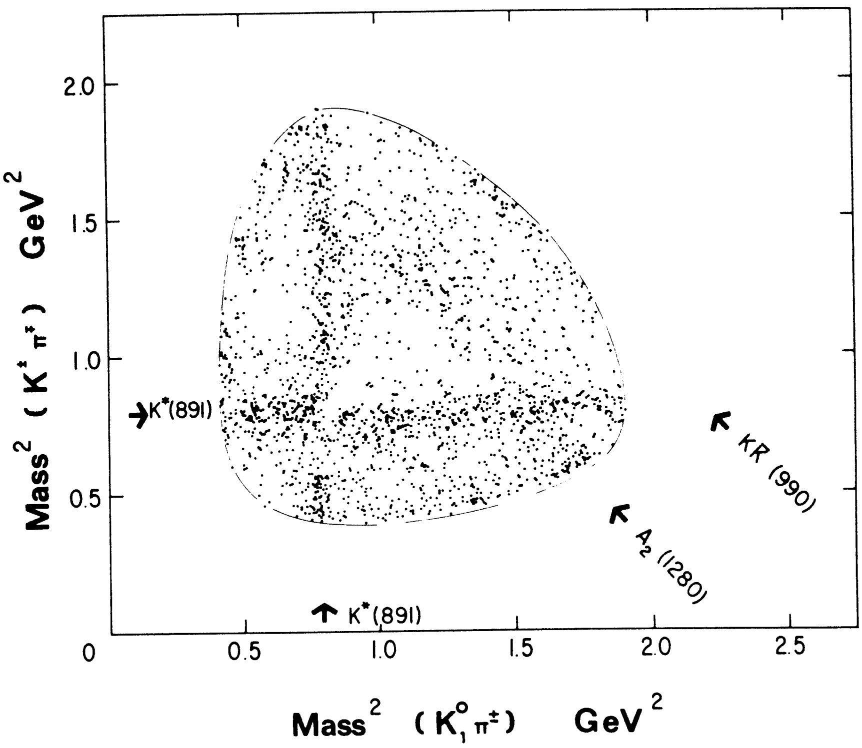


Fig. 25 Effective mass squared Dalitz plot for the reaction

$$\bar{p}p \rightarrow K_1^0K^\pm\pi^\mp$$

for annihilations at rest.

Four accumulations of events are visible: the $K^{*0}(891)$, $K^{*\pm}(891)$, $K\bar{K}(1280)$ and $K\bar{K}(990)$. They are indicated by the arrows[57].

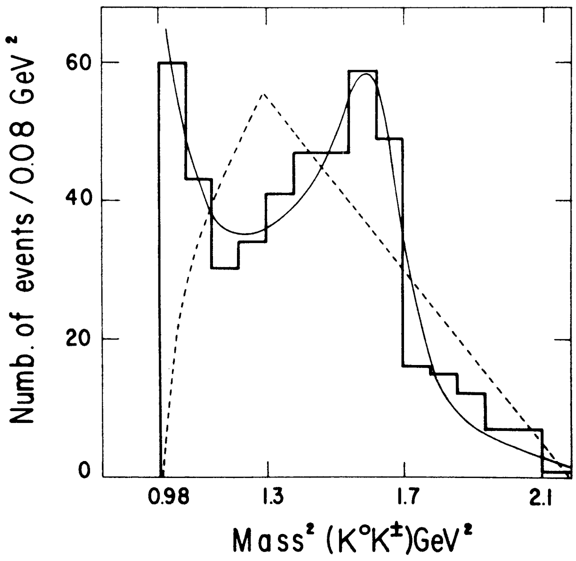


Fig. 26 ($K^oK^{\pm}$) effective mass squared spectrum for the reaction

$$\bar{p}p \rightarrow K_1^o K^{\pm} \pi^{\mp}$$

for annihilations at rest, when $M^2(K\pi) > 0.9$ GeV². [This cut is made to remove most of the K*(891) production[57]].

The full curve corresponds to the theoretical distribution as given by the best fit when two resonances $K\bar{K}(990)$ and $K\bar{K}(1280)$ are assumed to be produced.

The dotted curve corresponds to phase space.

The general features of this Dalitz plot are relatively well understood with the introduction of amplitudes corresponding to the production of K*(891), $A_2(1280)$, the presence of s-wave $K\pi$ amplitudes represented by small scattering lengths, but the ($K_1^oK^{\pm}$) enhancement at threshold can only be interpreted with the introduction of a genuine effect. To represent this effect, one may introduce an amplitude represented by a scattering length $1/(1-iaq)$, or a Breit-Wigner amplitude $1/(m^2_{K\bar{K}} - m_o^2 + iq\gamma)$(this form of the Breit-Wigner is more justified than the "constant width" formula $1/(m^2_{K\bar{K}} - m_o^2 + im_o\Gamma)$ since we want to describe an effect very near to threshold). Both representations give a good interpretation of the data: the s-wave scattering length is found to be equal to ± 2.0 f. The Breit-Wigner formula leads to the following assignments for the mass and width of this "new resonance".

$$m \sim 1016 \pm 10 \text{ MeV}$$
$$\gamma \sim 100 \pm 20 \text{ MeV}.$$

With these properties, and the probable quantum numbers $I^GJ^P = 1^-0^+$, this resonance cannot be identified with the resonances δ, X^o, H^o, etc. Its "natural" decay mode should be $\eta\pi$, but of course one cannot exclude the possibility that its coupling to $\eta\pi$ is very small.

A more attractive interpretation may be to consider the $K\bar{K}$ bound state which may correspond to the negative scattering length of 2 f [58]. To take into account the $\eta\pi$ channel, it is then worthwhile to introduce a complex scattering length a + ib: experimentally, b is found to be very small and suggests the existence of a narrow $\eta\pi$ resonance at 970 MeV.

If this interpretation is correct, it is tempting to identify this narrow $\eta\pi$ resonance with the relatively well-established $\delta^{\pm}$ meson (see Chapter 4.3), which has then the quantum numbers $I^G J^P = 1^- 0^+$ [57,59].

7.2 The S*(1050)

An enhancement has been observed in several reactions for the system $K_1^0 K_1^0$, presumably in I = 0, near 1050 MeV.

The observations come from several $\pi^{\pm}p$ reactions [Crennell et al.[60], Beusch et al.[61]] as well as from $\bar{p}p$ annihilations [Barlow et al.[62]]

The results of the $\bar{p}p$ annihilations at 1.2 GeV/c are particularly interesting since they show clearly the difference between the $(K_1^0 K^{\pm})$ enhancement discussed above (Section 7.1) and the $(K_1^0 K_1^0)$ behaviour. For this last system, the Breit-Wigner introduced for interpreting the mass spectrum gives a central mass value definitively above threshold:

$$M = 1045 \pm 9 \text{ MeV}, \quad \Gamma = 50 \pm 25 \text{ MeV}$$

in agreement with the results of Crennell et al., and Beusch et al.

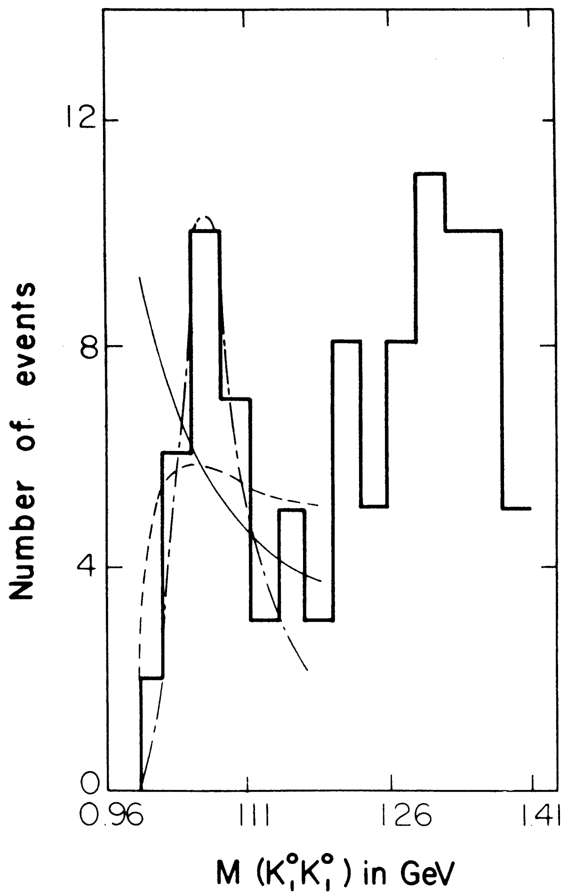


Fig. 27 $K_1^0 K_1^0$ effective mass spectrum for the reaction $\pi^- p \to K_1^0 K_1^0 n$ at 6 GeV. The dashed curve corresponds to an s-wave resonance; the dotted curve to an s-wave scattering length of $(\pm 1.4 + 0.2if)$, the solid curve to s-wave and scattering length of $(\pm 4 + 0.2if)$[60].

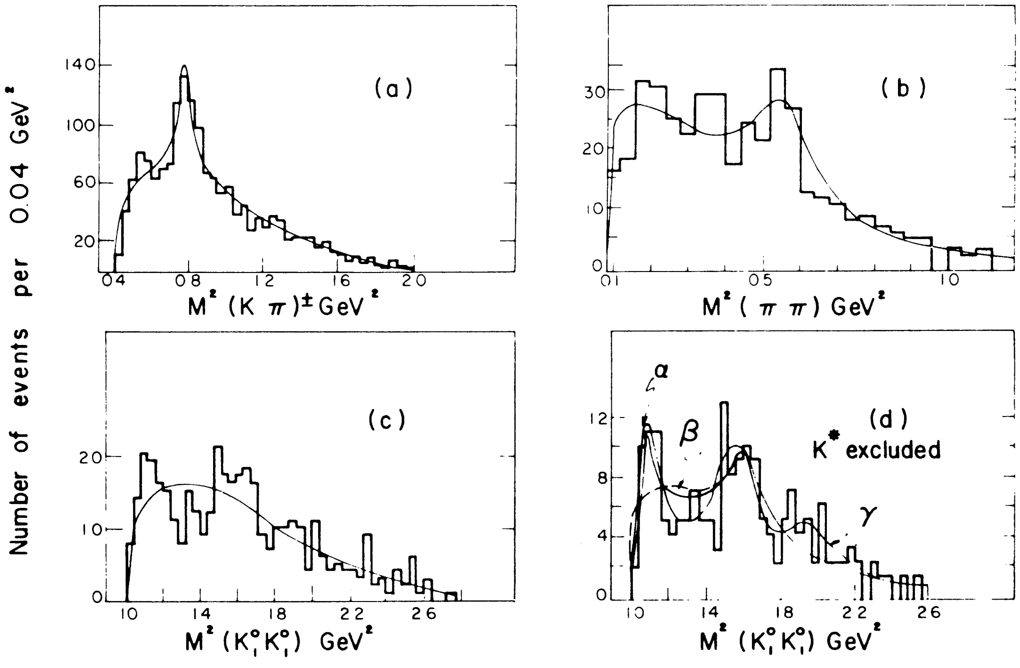


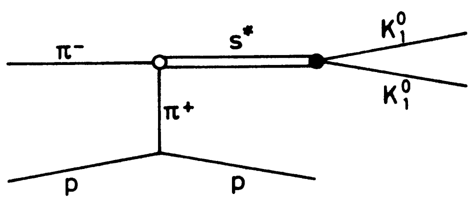
Fig. 28 ($K_1^0K_1^0$) effective mass spectrum for the reaction:

$$\pi^- p \rightarrow K_1^0 K_1^0 n \quad \text{at 12 GeV/c.}^{61)}$$

Apart from a large accumulation of events in the 1200-1400 MeV region, one observes a sharp peaking of events at 1070 MeV.

However, other experimental results still favour a large complex scattering length for the interpretation of this effect.

Assuming it is a resonance, with the quantum numbers $I^C J^P = 0^+ 0^+$, it could be observed in the ($\pi^+\pi^-$) system. No such resonance has yet been observed. This is somewhat surprising since some of the experiments which show the $K_1^0K_1^0$ enhancement are presumably due to the one-pion exchange mechanism according to the diagram:



which works only if s* is coupled to the ($\pi\pi$) system.

This model may also be taken as evidence for I = 0 for the s* since $C(K_1^0K_1^0) = +1$, $G(\pi^+\pi^-) = +1$ leads to even I.

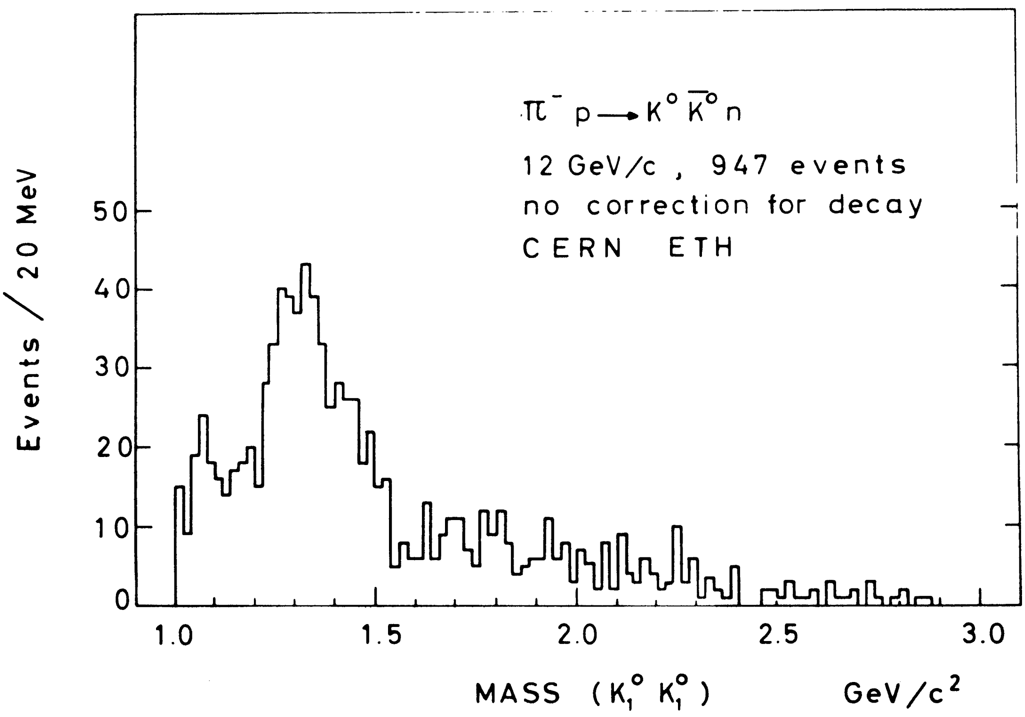


Fig. 29 $\bar{p}p \rightarrow K_1^0 K_1^0 \pi^+ \pi^-$ at 1.2 GeV/c [62].

These four histograms show the main features of the four-body annihilations: $\bar{p}p \rightarrow K_1^0 K_1^0 \pi^+ \pi^-$. In (a), one sees the abundant K*(891) production. In (b), there is some indication of ρ production. (c) and (d) deal with the $(K_1^0 K_1^0)$ effective mass spectrum. One sees two enhancements, at 1050 and 1300 MeV, in particular in (d) where the K*(891) events were removed. Curve α corresponds to the fit with two resonances, at 1050 MeV and 1300 MeV. Curve β corresponds to the fit to a scattering length and a resonance at 1300 MeV. Curve Υ corresponds to the fit to three resonances, at 1050, 1300 and 1470 MeV.

7.3 The f′(1500)meson

The enhancement observed in the $K_1^0 K_1^0$ system at $M \sim 1500$ MeV for the reaction

$$\pi^- + p \rightarrow \Lambda^0 + K_1^0 + K_1^0$$

has been called the f'^0 [63]. Its $K_1^0 K_1^0$ decay mode implies $J^P = 0^+, 2^+, 4^+ \ldots\ C = +1$.

But the isospin is not so well established. The absence of any charged $(K\bar{K})$ enhancement with the same mass suggests $I = 0$.

The decay angular distribution is not isotropic and agrees with the 2^+ assignment.

Another reason for rejecting the 0^+ assignment comes from the observation of the $(K\bar{K}\pi)$ decay mode (indeed $K^*\bar{K}$), but this observation is not so well established, since this channel is also open to the E^0-meson production which seems to interfere with the f′(1500).

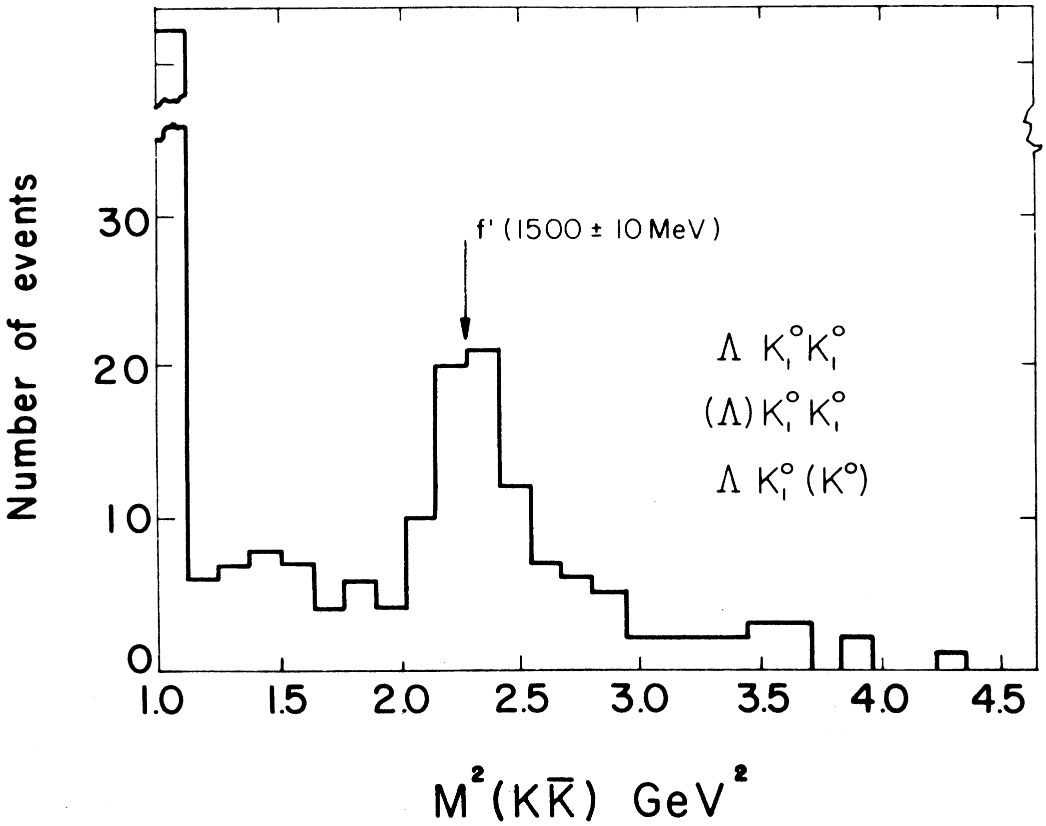


Fig. 30 $(K\bar{K})$ effective mass spectrum for the reaction

$$K^-p \to \Lambda K\bar{K} \quad \text{at } 4\text{-}5 \text{ GeV/c}.$$

The sharp peaking near threshold is due to the φ meson $\varphi \to K_1^0K_2^0$. The accumulation of events at 1500 MeV is taken as an evidence for a new meson, the f′(1500) [63].

It is found that this accumulation corresponds to a $(K_1^0K_1^0)$ system.

Although the f′(1500) could decay into a pair of pions, this decay mode has not been observed, and an upper limit for the branching ratio is:

$$\frac{\Gamma(\pi\pi)}{\Gamma(K\bar{K})} < 0.4\% \ .$$

Apart from this f'^0(1500), which has been observed until now in one reaction only (π^-p at 6 GeV/c), two experimental groups have observed a structure in the $K_1^0K_1^0$ around 1430 MeV [Beusch et al.[61], Barlow et al.[62]] which it is interesting to bring together with the apparent accumulation observed in the four-pion spectrum (when they form a pair of ρ mesons) by Cresti et al.[64].

8. MESONS WITH STRANGENESS ≠ 0

As for the dipion system, the $(K\pi)$ system presents two well-known resonances: the K*(891), with $J^P = 1^-$, and the K*(1410), with $J^P = 2^+$. They are both I = ½.

No scalar (Kπ) system has been observed with certitude, the existence of the κ(725) resonance being very doubtful nowadays.

On the other hand, several enhancements have been observed in the (Kπ) and (Kππ) systems in the neighbourhood of 1200-1300 MeV, and recently a beautiful peak has been reported near 1800 MeV.

We shall examine these experimental results one after the other, but let us first discuss the experimental situation of the K*(1410).

8.1 The K*(1410)

The K*(1410) was first observed by Haque et al.[65] in the three-body final state $\bar{K}^0\pi^-p$:

$$K^- + p \to \bar{K}^0 + \pi^- + p \quad \text{at } 3.5 \text{ GeV/c} .$$

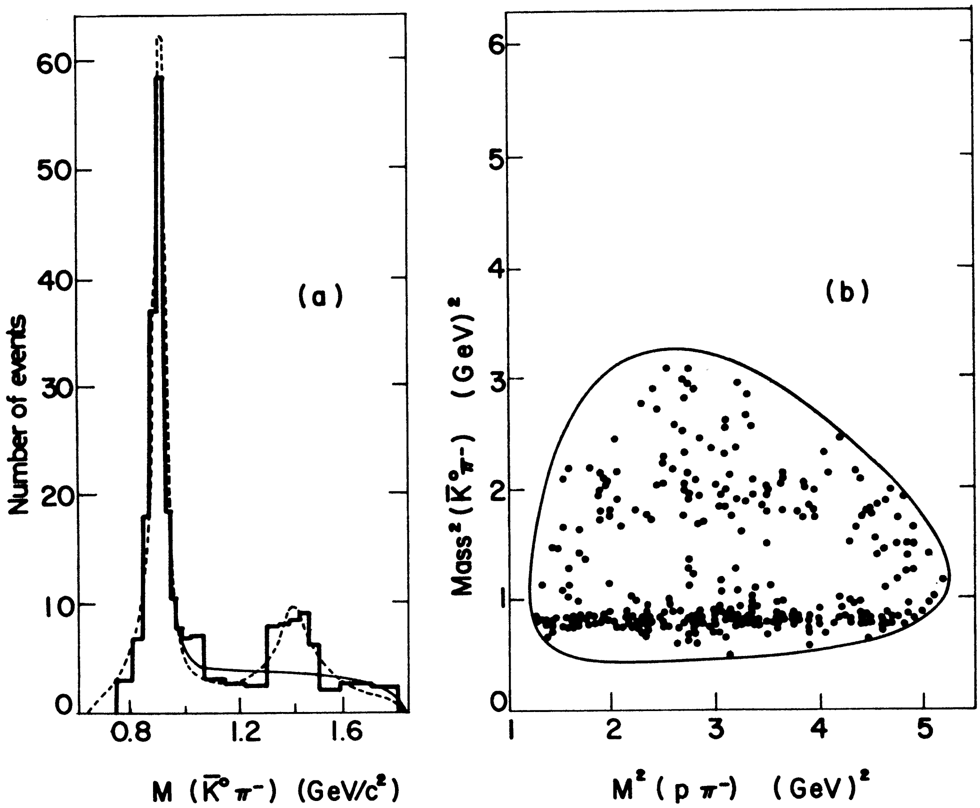


Fig. 31 (Kπ) effective mass spectrum for the reaction

$$K^-p \to \bar{K}^0\pi^-p \quad \text{at } 3.5 \text{ GeV/c} .$$

In addition to the K*(891) enhancement, an accumulation of events appear at 1400 MeV.

The dotted curve corresponds to K*(891) production plus phase-space. The solid curve corresponds to K*(891) and K*(1400) production plus phase-space.

The Dalitz plot shows the absence of significant N* production[65].

It has also been observed in the reaction:

$$\pi^- + p \rightarrow \Lambda + \pi + K \quad \text{at 4 GeV/c.}$$

The isospin $I = \frac{1}{2}$ is suggested by the absence of any enhancement in the $I = \frac{3}{2}$ channel.

The spin-parity analysis has been achieved with the use of the angular distributions parametrized in terms of the density matrix elements as described in Section 3.3: the results slightly favour 2^+ over 1^-.

The K*(1410) has been observed to decay into a two-body, $K^* \rightarrow K\pi$ ($\sim$ 50%), but also into

$$K^*(1410) \rightarrow K^*(891)\pi \sim 40\%$$

and

$$K^*(1410) \rightarrow K\rho \quad \sim 10\% \, .$$

There is still some disagreement between the measurements of the width (and, to a lesser extent, of the mass) of the K*(1410).

A weighted average gives:

$$M = 1411 \pm 6 \text{ MeV}$$
$$\Gamma = 92 \pm 7 \text{ MeV} \, .$$

8.2 The κ(725)

There has always been a certain disagreement between the different experimental results on the $(K\pi)$ enhancement observed around 725 MeV, first as a narrow peak (Γ < 25 MeV) but later as a broad enhancement ($\sim$ 50 MeV).

The evidence for a narrow peak [Alexander et al.[66]), Wojcicki et al.[67])] has disappeared with more statistics [Lynch[68])].

The nature of the broad enhancement sometimes observed in the $K\pi$ system around 700 MeV is unknown, but since it is often associated with the presence of other resonances in other channels [the N*(1238), for instance] it may be the effect of strong interferences between the $T = \frac{1}{2}$ $K\pi$ s-wave and other amplitudes.

The study of the three-body annihilations at rest, $\bar{p}p \rightarrow K_1^0 K^{\pm} \pi^{\mp}$, shows the $T = \frac{1}{2}$ $K\pi$ s-wave may be represented by a scattering length of 0.3 fermi near the threshold of the $(K\pi)$ system.

In conclusion, there is no clear evidence, for the time being, for a $(K\pi)$ resonance at 725 MeV.

8.3 The 1180-1320 MeV mass region

Many enhancements have been reported for the $(K\pi)$ and $(K\pi\pi)$ systems in the 1180-1320 MeV mass region.

Three different groups have reported a peak at 1175 ± 15 MeV [Wangler et al.[69]), Miller et al.[70]), Bishop et al.[71])], but there are at least as many negative results coming from very similar experiments.

At 1230 MeV, a broad enhancement has been observed for the $(K\pi\pi)$ in the four-body annihilations at rest:

$$\bar{p}p \to K_1^0 K_1^0 \pi^+ \pi^-$$
$$\bar{p}p \to K_1^0 K_2^0 \pi^+ \pi^-$$
$$\bar{p}p \to K_1^0 K^\pm \pi^\mp \pi^0 \ .$$

This enhancement (sometimes called the C meson), is statistically very significant (the authors attribute more than 50% of the events to this effect).

But its nature is not well understood: on the one hand, it seems very difficult to interpret it as a reflection of the ρ^0 or K*(891) mesons (or both) which appear to be abundantly produced in these reactions. On the other hand, it does not show uo in the four-body $\bar{p}p$ annihilations at 1.2 GeV/c.

However, the C meson may be one of the manifestations of a K* resonance with a mass of the order of 1250 MeV, since many experimental results could be better understood if such a resonance was present.

In fact, many experimental results speak in favour of a broad $(K\pi\pi)$ enhancement in the 1250-1320 MeV region, but there are several indications which suggest that there are perhaps two resonances in this region [in addition to the K*(1410)].

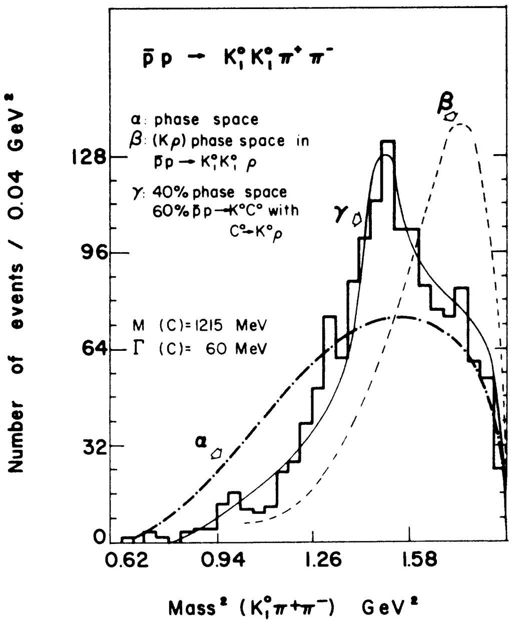


Fig. 32 $(K_1^0\pi^+\pi^-)$ effective mass spectrum for the reaction

$\bar{p}p \to K_1^0 K_1^0 \pi^+ \pi^-$ (annihilations at rest).

The accumulation of events at $M^2 \sim 1.52$ GeV^2 cannot be explained by the reflection of the ρ meson (curve β). A good fit is obtained when a $(K\pi\pi)$ resonance is introduced at M = 1230 MeV.

One resonance could be centred at ~ 1250 MeV, whereas the second one could be the K*(1320) resonance proposed by Almeida et al.[72].

The 3 GeV/c K^+p experiment performed at CERN[73] shows a peak in the (Kππ) system at 1280 MeV. It may be different from the 1320 MeV peak observed by the same group, but at a higher energy, and by Shen et al.[74].

The decay into Kπ is not observed, but it is not completely excluded. It would, of course, be interesting to study the variation of the branching ratio Kπ/Kππ as a function of the energy.

It will probably be very difficult to get a clear understanding of the whole situation, since these resonances seem to have rather large widths (~ 60 MeV) and the background, as for the A_1(1080) meson, may have a structure which is characteristic of the Deck mechanism [with peripheral production of the K*(891) meson].

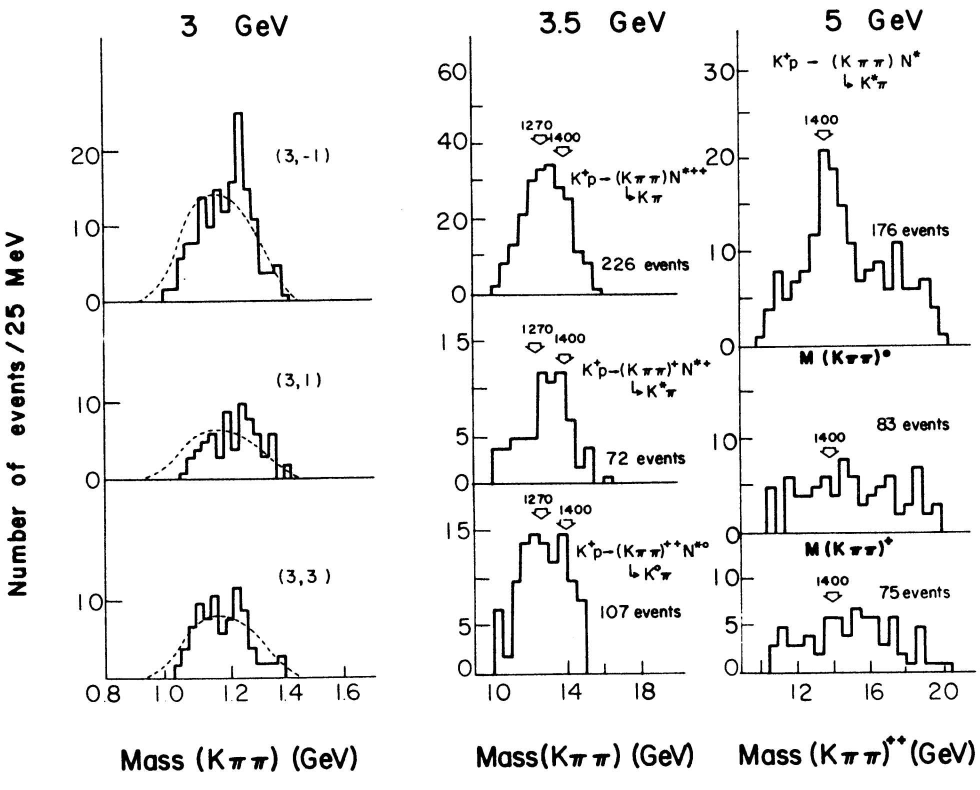


Fig. 33 (Kππ) effective mass spectra for the reaction

$$K^+p \rightarrow Kp\pi\pi,\ Kn\pi\pi$$

at 3, 3.5 and 5 GeV/c.[73].

The contributions from different charge states are given. The enhancement observed at 1280 MeV does not appear in the T = 3/2 histograms.

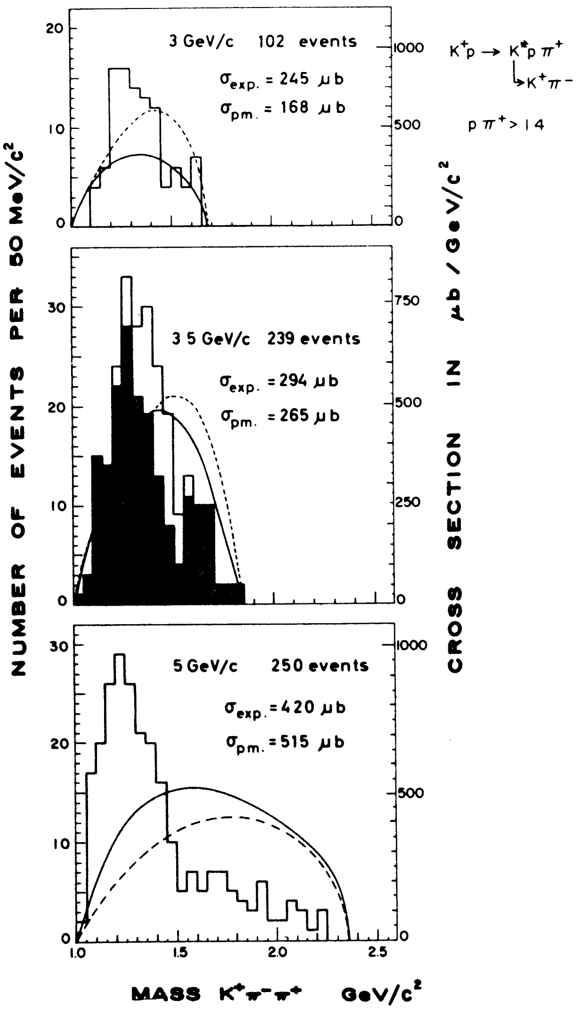


Fig. 34 $(K^+\pi^-\pi^+)$ effective mass distributions from the reaction

$K^+p \to K^*(891)p\pi^+$ at 3, 3.5 and 5 GeV/c,

N^* excluded.

The dotted curves represent the phase-space (taking into account the cuts introduced to remove the N^*).

The solid line represents an attempt to take into account the "peripheral" character of the production.

None of these curves reproduces the experimental situation. The enhancement in the 1200 - 1400 MeV mass region is therefore taken as evidence for one, if not two, $(K\pi\pi)$ resonances.

At 3.5 GeV/c, the $K^*(1400)$ has been observed to decay into a $(K\pi)$ combination, in the reaction

$$K^+p \to K\pi p \ .$$

Using a branching ratio $K^*(1400) \to K^*\pi/K^*(1400) \to K\pi = 0.6$, it is possible to estimate the contribution of this $K^*(1400)$ in the $(K\pi\pi)$ spectrum. The left-over events are shaded and peak at 1270 MeV[73].

An experiment done at CERN with 10 GeV/c K^- also shows a broad accumulation of events around 1300 MeV with a width larger than 150 MeV. It suggests again the existence of two unresolved resonances [since the width of the K*(1320) is of the order of 60 MeV].

In these conditions, it is, of course, very difficult to get an unambiguous result for a spin parity analysis of these states. If the $K\pi$ decay mode is confirmed (at least for one of these objects), it implies $P = (-1)^J$. For the time being, the experimental results rather suggest the reverse conclusion [the absence of $K\pi$ decay mode is taken as weak evidence for $P = (-1)^{J+1}$].

8.4 The K*(1800)

Recently, the Aachen - Berlin - CERN - London (Imperial College) - Vienna Collaboration[75] has reported the existence of a new meson, which they call the L meson. It is observed in the $K^*\pi$, $K\rho$, and $K\omega$ system (but not in the $K\pi$ system) at 1790 MeV with $\Gamma \sim 80$ MeV.

The observation of $L \to K\omega$ leads to $I(L) = \frac{1}{2}$.

The absence of the $K\pi$ decay mode suggests that $P(L) = (-1)^{J(L)+1}$.

9. OTHER RESONANCES

9.1 The B(1200)

Several experimental results have recently cast some doubts on the existence of the B(1200) meson (an enhancement observed in the $\omega\pi^\pm$ system at 1200 MeV with $\Gamma \sim 90$ MeV) [Goldhaber et al.[76], Chung et al.[77]].

In fact, many experiments have had negative results with regard to the B meson. In particular, no B meson is observed in $\bar{p}p$ annihilations into seven pions, where the ω^0 meson is copiously produced at 1.2 GeV/c.

However, a recent experiment on five-pion $\bar{p}p$ annihilation at rest brings new positive evidence for the B meson [Baltay et al.[78]]

$$\bar{p}p \to B^\pm\pi^\mp \to \omega^0\pi^+\pi^- \ .$$

Although this evidence seems to be quite convincing, the detailed analysis of the reaction may be rather difficult, since an abundant production of ρ is not excluded:

$$\bar{p}p \to \omega^0\rho^0 \ .$$

The search for $B \to K\bar{K}$ and $B \to K\bar{K}\pi$ in annihilations at rest has been unsuccessful. The absence of $B \to K\bar{K}$, $B \to \pi\pi$ suggests that the spin parity of the B meson may be of the series $P = (-1)^{J+1}$.

9.2 The D(1280)

Two experiments have led to the observation of a rather sharp enhancement in the neutral ($K\bar{K}\pi$) system at 1280 MeV [Miller et al.[79], d'Andlau et al.[80]].

Miller et al. studied the reaction

$$\pi^- + p \to K^\pm + K_1^0 + \pi^\mp + n \ ,$$

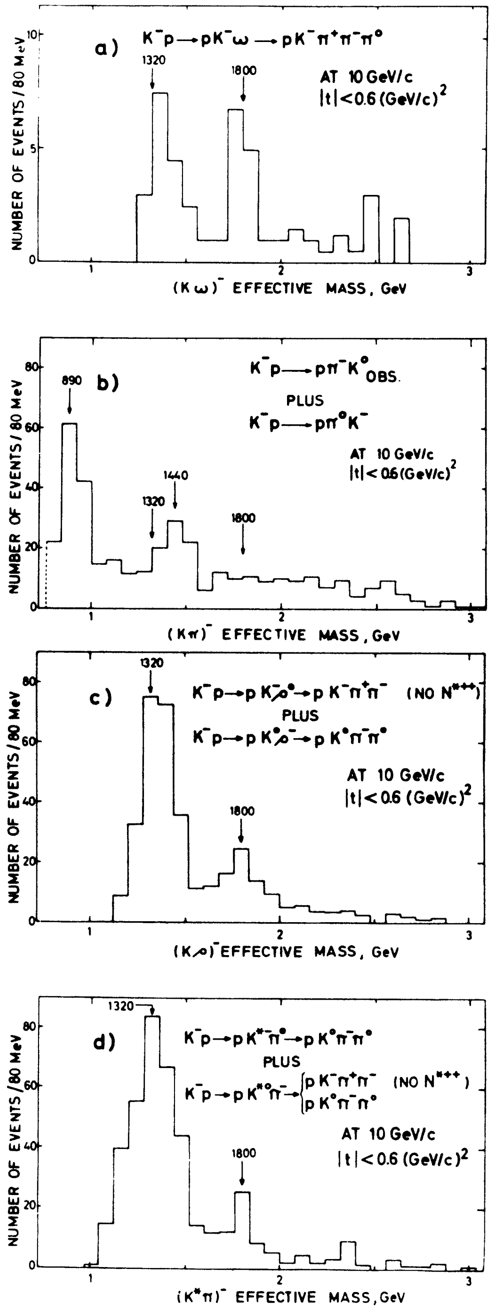


Fig. 35 The L-meson[75],

a) ($K\omega$) effective mass spectrum. A peak is observed at 1800 MeV.

b) ($K\pi$) effective mass spectrum. No peak is observed at 1800 MeV, whereas the K*(891) and K*(1400) are clearly produced.

c) ($K\rho$) effective mass spectrum; the L(1800) is visible.

d) ($K^*\pi$) effective mass spectrum; the L(1800) is again visible.

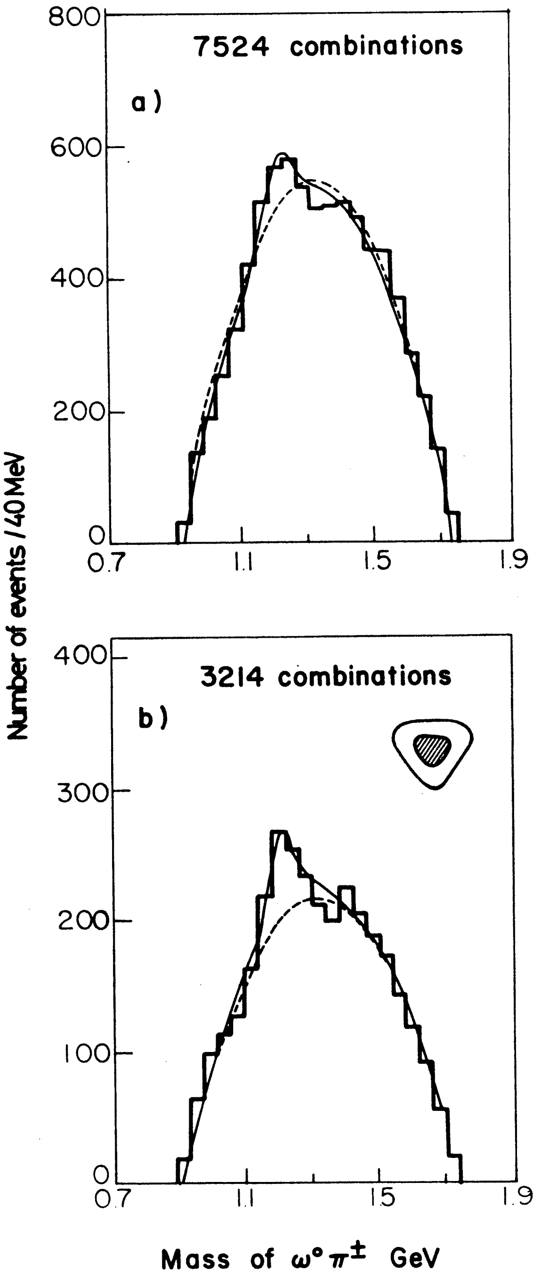


Fig. 36 The B meson[78].

a) Distribution in the effective mass of the $\omega^{\circ}\pi^{\pm}$ in the reaction $\bar{p} + p \rightarrow \omega^{\circ} + \pi^{+} + \pi^{-}$. The dashed curve represents the background expected from the fit to the $\omega^{\circ}\pi^{+}\pi^{-}$ Dalitz plot with the B-meson bands removed; the curve has been normalized to the total number of events in the distribution. The solid curve is the best fit including B-meson production.

b) $\omega^{\circ}\pi^{\pm}$ mass distribution selecting events in which the ω° lies in the central region of the ω° decay Dalitz plot. The solid curve is the best fit including B-meson production discussed in the text. The dashed background curve here has been normalized to the experimental distribution outside of the B-meson region.

whereas d'Andlau et al. studied the annihilations:

$$\bar{p}p \to K_1^0 K^\pm \pi^\mp \pi^+ \pi^-$$

and

$$\bar{p}p \to K_1^0 K^\pm \pi^\mp \pi^+ \pi^- \pi^0 \text{ at } 1.2 \text{ GeV/c.}$$

The quantum numbers of the D meson are not well known.

Let us assume that the accumulation of the $(K\bar{K})$ mass spectrum near threshold for events associated to the D meson is an indication that the orbital angular momentum of the $(K\bar{K})$ system is 0; then

$$G(K\bar{K}) = -1 \quad [\text{since } I(K\bar{K}) = 1]$$

and

$$G(D) = G(K\bar{K}) \cdot G(\pi) = +1 \quad .$$

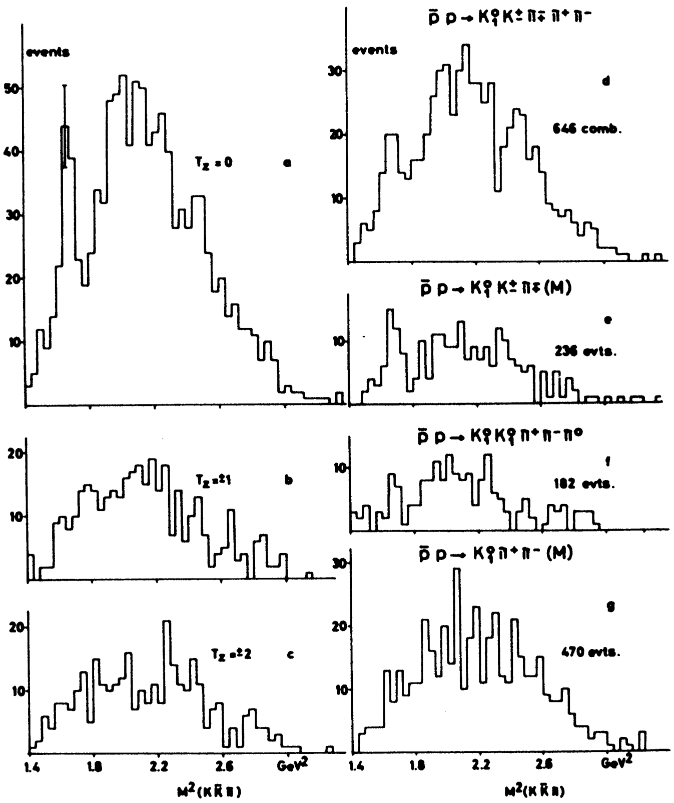


Fig. 37 $(K\bar{K}\pi)$ effective mass spectra for the reactions:

1) $\bar{p}p \to K_1^0 K^\pm \pi^\mp \pi^+ \pi^-$

2) $\bar{p}p \to K_1^0 K_1^0 \pi^+ \pi^- \pi^0$

3) $\bar{p}p \to K_1^0 K^\pm \pi^\mp \pi^0 \pi^0$

4) $\bar{p}p \to K_1^0 \pi^+ \pi^- M$ [80].

(a), (b), (c) correspond to the neutral, singly, and doubly charged systems. A peak is observed in the neutral one at 1280 MeV: the D^0 meson. (d), (e), (f), (g) correspond to the neutral $(K\bar{K}\pi)$ and (KM) systems for the reactions 1), 2), 3), 4), respectively.

d'Andlau et al. have also observed the decay mode:

$$D \rightarrow K_1^0 K_1^0 \pi^0$$

(but with a very weak statistical significance: 8 ± 4 events). If this decay mode is present:

$$C(D) = 1$$

and

$$I(D) = 0 \ .$$

For the spin parity determination, the same assumption for the orbital angular momentum state of the $(K\bar{K})$ system leads to the relation:

$$P(D) = (-1)^{J(D)+1} \ .$$

The decay Dalitz plot favours 1^+ and 2^-, unless an additional $(K\bar{K})$ genuine effect is introduced, in which case one cannot exclude $J^P = 0^-$ for the D meson.

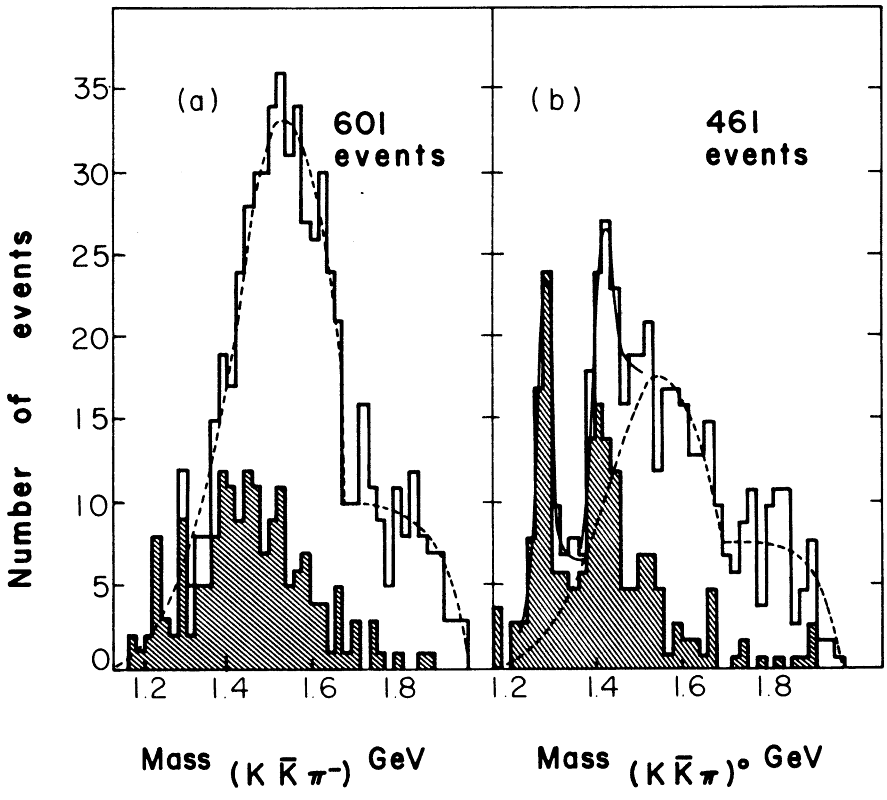


Fig. 38 $(K\bar{K}\pi)$ effective mass spectra for the reaction:

$$\pi^- p \rightarrow K\bar{K}\pi p \text{ at } 1.7 - 4.2 \text{ GeV/c } ^{79)} \ .$$

(a) corresponds to the charged $(K\bar{K}\pi)$ combinations, whereas (b) corresponds to the neutral ones. One observes two enhancements in the neutral spectrum, at 1280 and 1420 MeV. They correspond to the D^0 and E^0 mesons. These accumulations are enhanced when a selection of events is made: $M^2(K\bar{K}) < 1.1 \text{ GeV}^2$.

9.3 Higher mass resonances

The "missing-mass"spectrometer" experiment carried out at CERN has led to the observation of many enhancements in the charged boson system produced in $\pi^- p$ reactions of several energies.

In addition to large peaks corresponding to the ρ and A_2 mesons, several "bumps" are observed:

at 962 MeV: the δ discussed in Section 4.3;
at 1632, 1700 and 1748 MeV: the R meson, one of these "bumps" being perhaps the manifestation of the g meson discussed in Section 5.3;
at 1929, 2195, and 2382 MeV: (S, T, U mesons) .

It is important to confirm these results experimentally.

One possibility (at least for masses larger than 1900 MeV) is to study the $\bar{p}p$ annihilations as a function of the incident energy: when the total energy in the c.m. system corresponds to the mass of a boson, one may expect to observe a fluctuation in the channels coupled to this boson. The total $\bar{p}p$ cross-section already shows some fluctuations which could be due to the excitation of heavy bosons.

Other experimental techniques have also produced evidence for heavy bosons: for instance, the analysis of the multipion $\bar{p}p$ annihilations [(French et al.[81])] indicates the existence of "enhancements" in the $\omega\pi\pi$ system at 1690 and 1848 MeV; other bubble chamber experiments have also led to the observation of "enhancements" in the $\rho\pi\pi$ system at 1717 and 1832 MeV. Some of these enhancements are probably identical to some of the peaks observed in the missing-mass spectrometer experiment. It is too early to attempt such an identification.

10. CONCLUSIONS

In opposition to the feeling one may get from the attractive single models proposed recently for the mesons, the experimental situation is still far from being satisfactory.

In the past few years, the number of resonances observed in practically all possible two-body and three-body combinations of mesons has kept increasing. However, one is still far from knowing the quantum numbers of most of these resonances. Sometimes, even their "nature" is doubtful.

Although there is yet no clear evidence for $T = 3/2$, 2 and $S > 1$ resonances, it is certainly much too early to deduce any firm conclusion from this absence which could very well be a temporary situation.

Even the nonet of "pseudoscalar mesons" (π, K, η, η') may be subject to a dramatic revision (if the η' or X^0 is not an isoscalar). However, in this particular case, there are very good reasons to keep the η' in this nonet and wait until we have a better knowledge of the "$\delta^{\pm}$ mesons". However, we have mentioned in Section 4.4 that the E^0(1420) could very well be another pseudoscalar meson, in which case we are in presence, not of a nonet, but of a decuplet of pseudoscalar mesons.

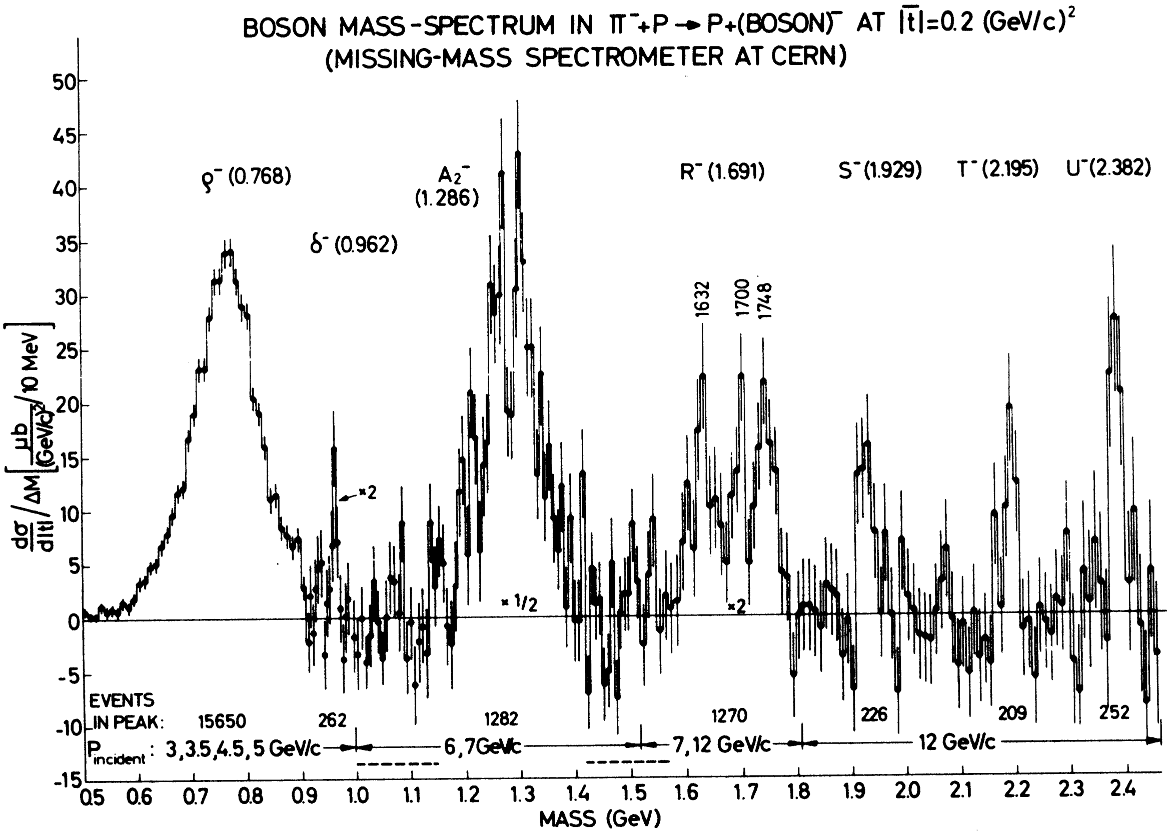


Fig. 39 Results of the "missing-mass spectrometer experiment" done at CERN, where many new peaks have been observed, at 962, 1632, 1700, 1748, 1929, 2195, and 2382 MeV.

The nonet of vector mesons is in a good shape (ρ, K^*, ω, φ).

Quite some time ago it was proposed to build a nonet of $J^P = 2^+$ mesons with the A_2 (1300), the $K^*(1410)$, the f^0, and f'^0. There is very little doubt that a 2^+ meson is present, with a I = 1 component near 1300 MeV (the $K\bar{K}$ system provides plenty of evidence for it). The $K^*(1410)$ has probably $J^P = 2^+$ (although 1^- is not completely excluded) but the f^0, and especially the f'^0, are not so well understood.

More information on the f'^0 is certainly needed, but it fits in so well with the 2^+ nonet that it is reasonable to leave it here as long as no serious difficulty is brought about by new experimental results.

Assuming that these three nonets (0^-, 1^-, 2^+) are taken for granted, we are left with a dozen resonances which may be candidates for scalar, pseudovectors, 2^- mesons.

For the scalar mesons ($J^P = 0^+$), we have seen that there is yet no compelling evidence for a $\sigma(400)$ or an $\epsilon(700)$ isoscalar object. But it is not excluded that the $K_1^0K_1^0$ enhancement observed first above threshold ($\sim$ 1050 MeV) is a good $J^P = 0^+$ candidate (with, probably, I = 0). For the I = ½ member of a scalar nonet, we have seen that the K(725) has vanished with more data and we have no serious candidate to replace it.

For the I = 1 member, the $K\bar{K}$ enhancement observed at threshold could eventually correspond to a resonance $I^GJ^P = 1^-0^+$.

For the 1^+ mesons, one may consider: the $H^0(975)$ and the B(1200), which are both C = -1; the $A_1(1080)$ and the D(1280) which are probably C = +1. For the T = ½ components, we may have two K^* around 1250 and 1320 MeV which are difficult to disentangle. Furthermore, it is not completely excluded that two mesons with I = 1 are present in the $\rho\pi$ and $K\bar{K}$ systems around 1300 MeV (if this structure was confirmed for $K\bar{K}$ it would, of course, exclude the 1^+ assignment).

Finally, heavier mesons have recently been observed in the ($\pi\pi$), ($\pi\pi\pi$), ($K\pi\pi$), ($\omega\pi\pi$), ($\rho\pi\pi$ systems) [g(1640), A_3(1650), L(1800), R, S, T, U]. We are far from being able to assign quantum numbers to these states.

* * *

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